



# Life on Earth – an Accident ?

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

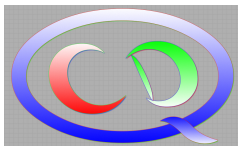
supported by DFG, SFB/TR-110

by CAS, PIFI

by VolkswagenStftung

by ERC, EXOTIC

by NRW-FAIR



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- Summary & outlook

# The anthropic principle

# The Anthropic Principle (AP)

- so **many** parameters in the Standard Model, the landscape of string theory, . . .

⇒ The anthropic principle:

“The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirements that the Universe be old enough for it to have already done so.”

Carter 1974, Barrow & Tipler 1988, . . .

⇒ can this be tested? / have physical consequences?

- Ex. 1: “Anthropic bound on the cosmological constant” Weinberg (1987) [1036 cites]
- Ex. 2: “The anthropic string theory landscape” Susskind (2003) [1126 cites]

# A prime example of the AP

- Hoyle (1953):

Prediction of an excited level in carbon-12 to allow for a sufficient production of heavy elements ( $^{12}\text{C}$ ,  $^{16}\text{O}$ ,...) in stars

- was later heralded as a prime example for the AP:

“As far as we know, this is the only genuine anthropic principle prediction”

Carr & Rees 1989

“In 1953 Hoyle made an anthropic prediction on an excited state – ‘level of life’ – for carbon production in stars”

Linde 2007

“A prototype example of this kind of anthropic reasoning was provided by Fred Hoyle’s observation of the triple alpha process...”

Carter 2006

# The relevant question

Date: Sat, 25 Dec 2010 20:03:42 -0600  
 From: Steven Weinberg <weinberg@zippy.ph.utexas.edu>  
 To: Ulf-G. Meissner <meissner@hiskp.uni-bonn.de>  
 Subject: Re: Hoyle state in  $^{12}\text{C}$

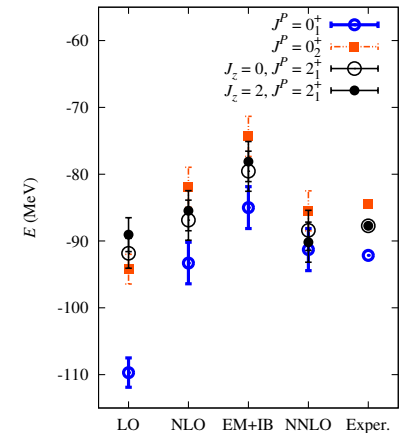
Dear Professor Meissner,

Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in  $^{12}\text{C}$ , but also of the ground states of  $^4\text{He}$  and  $^8\text{Be}$ . How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of  $^4\text{He}$  and  $^8\text{Be}$  to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of  $^8\text{Be}$  and  $^4\text{He}$ .

All best,

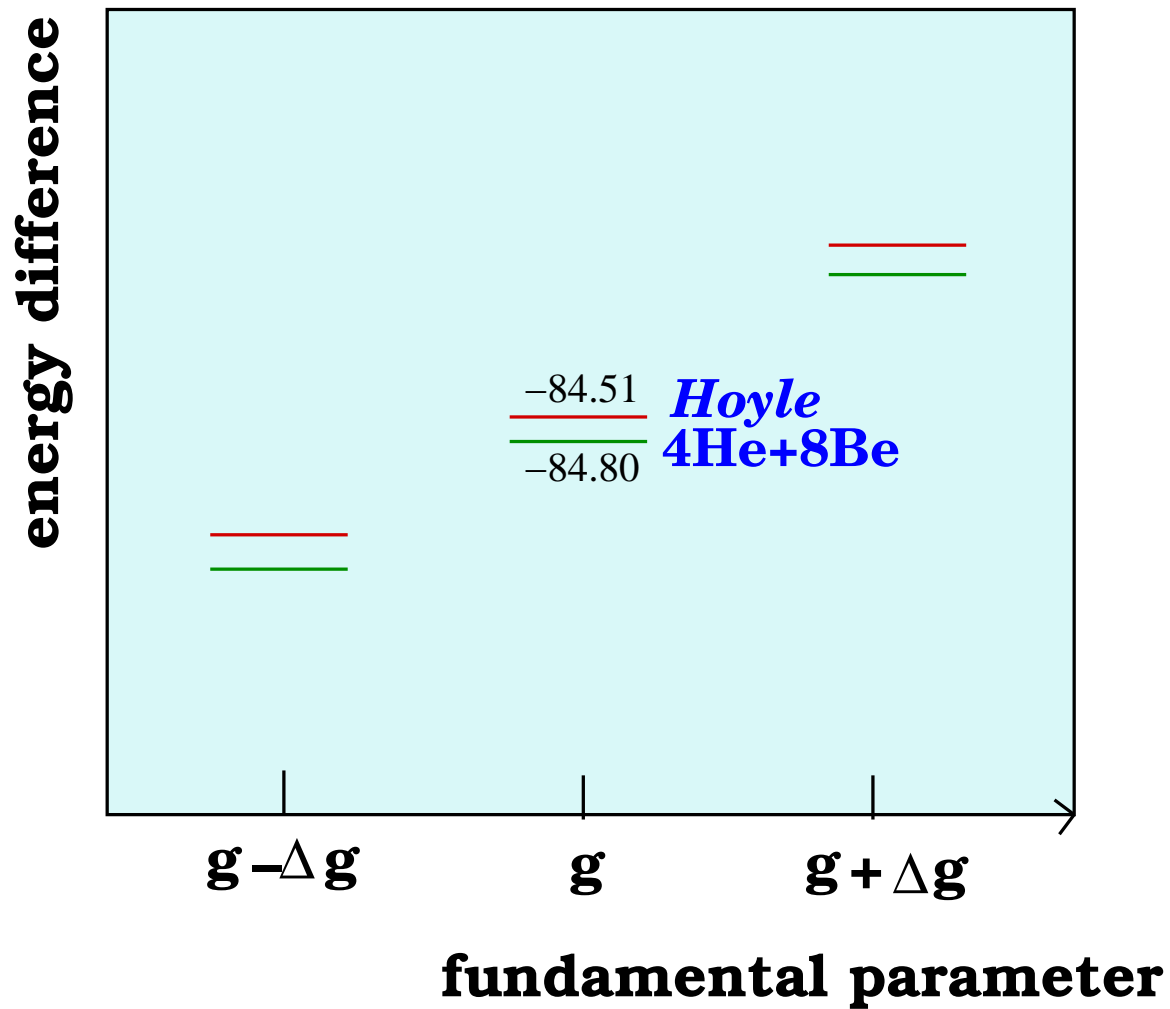
Steve Weinberg

- How does the Hoyle state move relative to the  $4\text{He}+8\text{Be}$  threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*



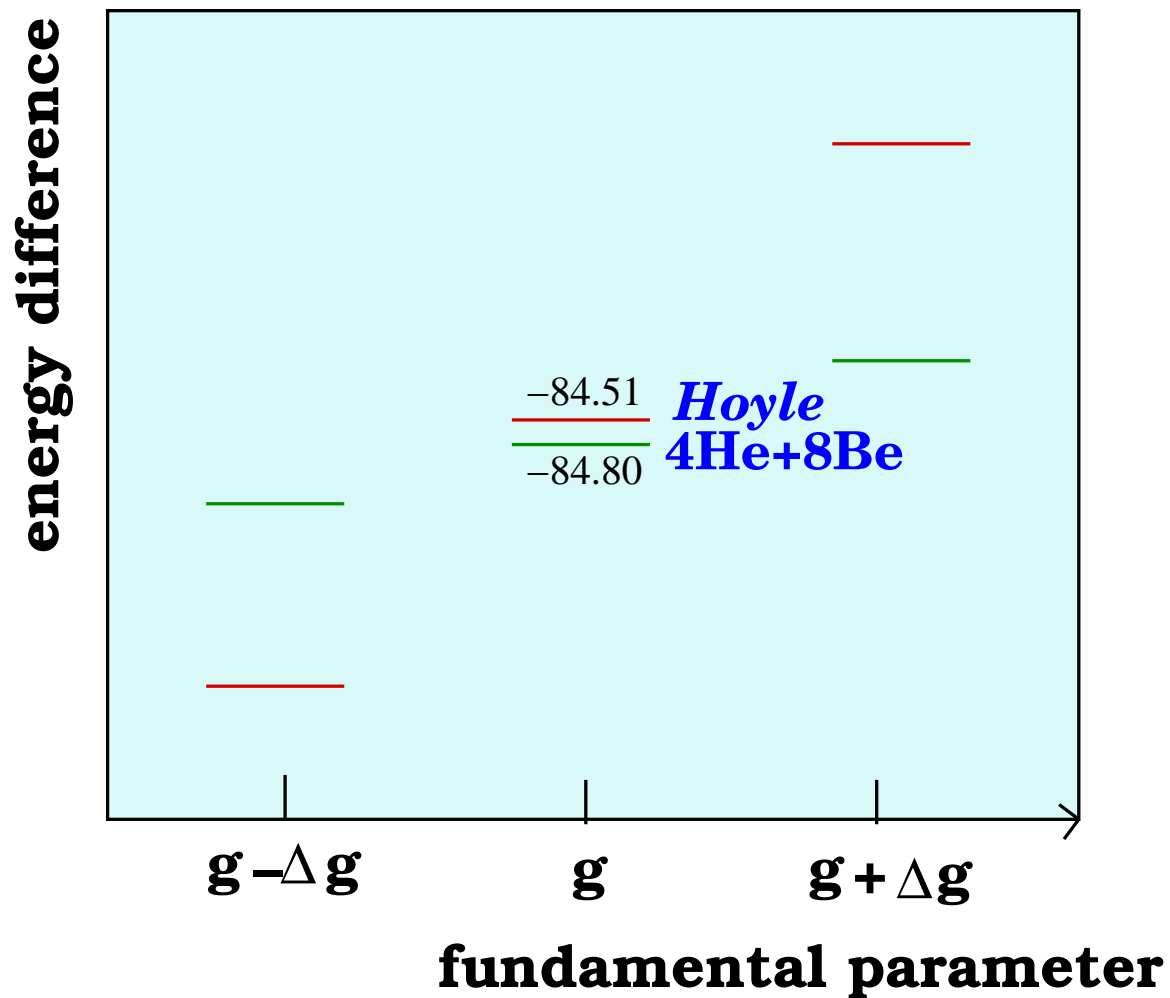
# The non-anthropropic scenario

- Weinberg's assumption: The Hoyle state stays close to the  $4\text{He}+8\text{Be}$  threshold



# The anthropic scenario

- The AP strikes back: The Hoyle state moves away from the  $4\text{He}+8\text{Be}$  threshold









# Definition of the physics problem



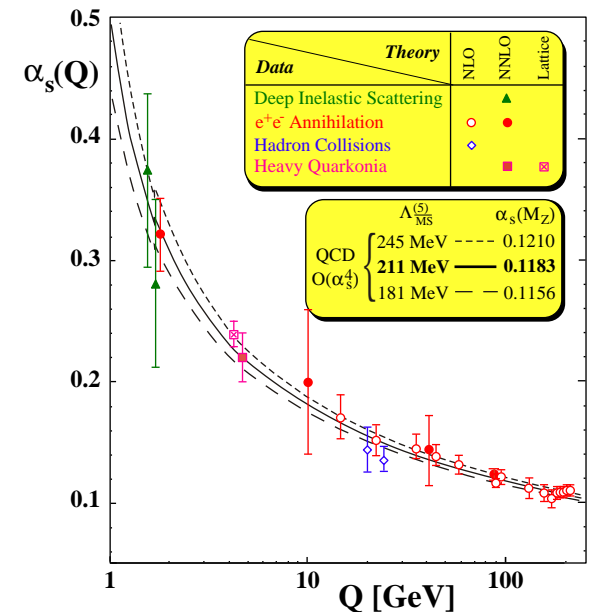


# Emergence of structure in QCD

- The strong interactions are described by QCD:

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{q}_f (i\gamma_\mu D^\mu - m_f) q_f + \dots$$

- **up** and **down** quarks are very light, a few MeV
  - Quarks and gluons are confined within **hadrons**
  - Protons and neutrons form **atomic nuclei**
- ⇒ This requires the inclusion of electromagnetism described by QED with  $\alpha_{EM} \simeq 1/137$



[from S. Bethge]

So how sensitive are these strongly interacting composites to variations of the fundamental parameters of QCD+QED?  
or: how accidental is life on Earth?

# Quark mass dependence of the nuclear forces

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,  
Phys. Rev. D **87** (2013) 085018



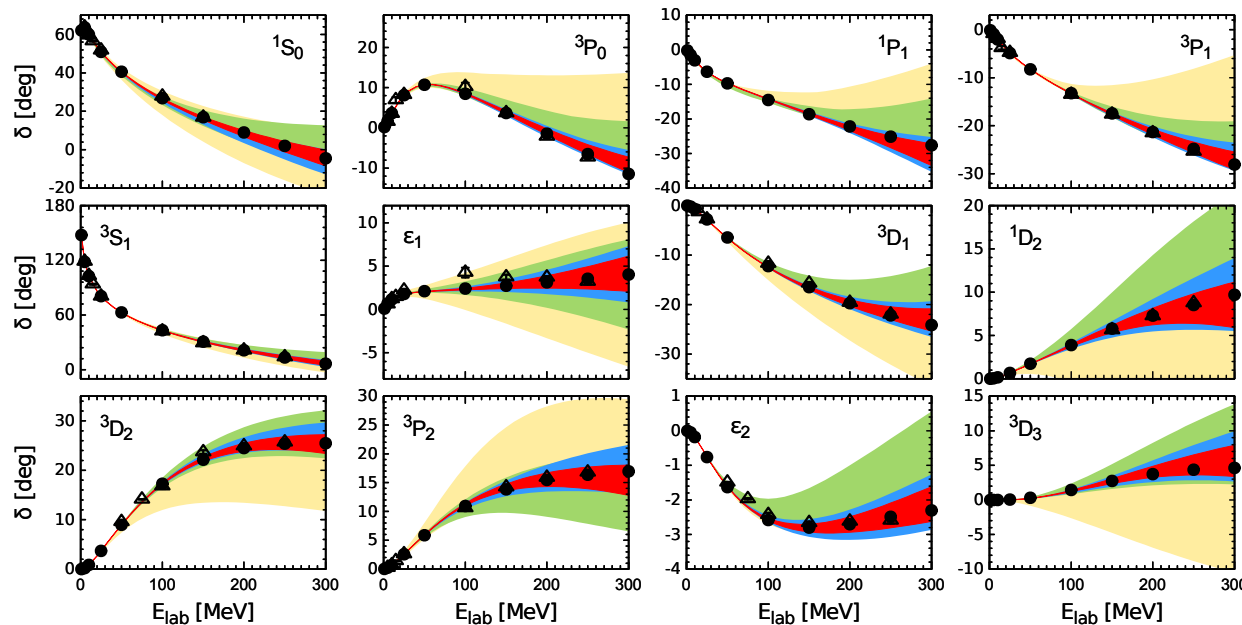


# Chiral nuclear EFT: Results

- Expansion to fifth order in the chiral expansion [Weinberg's power counting]

Epelbaum, Krebs, UGM, Phys.Rev.Lett. **115** (2015) 122301; Eur. Phys. J. A **51** (2015) 53

- phase shifts

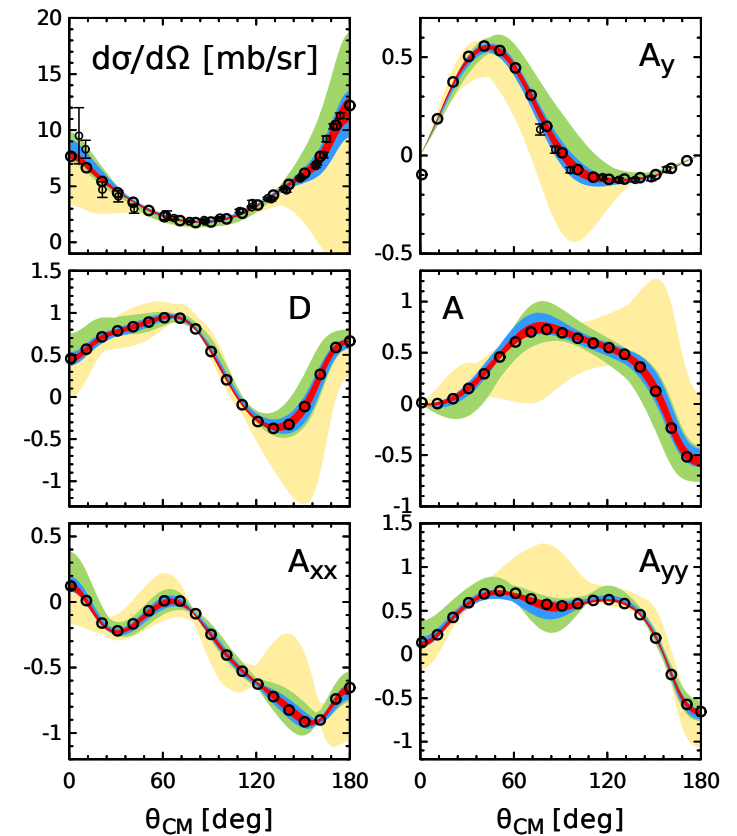


NLO N2LO N3LO N4LO

⇒ now a precision tool in nuclear physics

see e.g. Epelbaum, Krebs, Reinert, Front. in Phys. **8** (2020) 98

- np scattering at 200 MeV



# Quark mass dependence of hadron masses etc

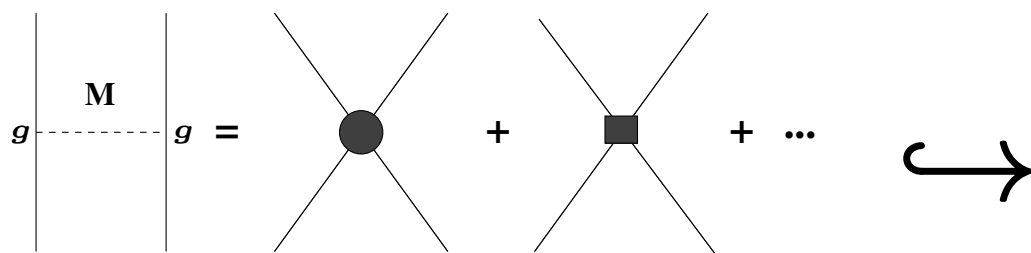
- Quark mass dependence of hadron properties:

$$\frac{\delta O_H}{\delta m_f} \equiv K_H^f \frac{O_H}{m_f}, \quad f = u, d, s$$

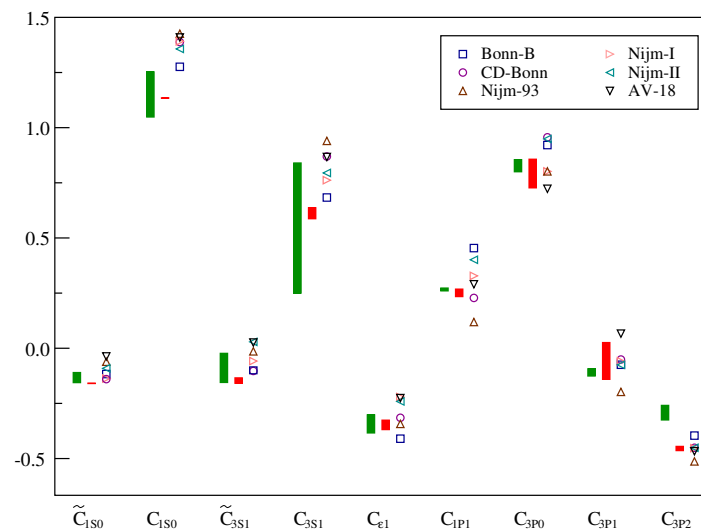
- Pion and nucleon properties from lattice QCD combined with CHPT
- Contact interactions modeled by heavy meson exchanges + unitarized CHPT

Epelbaum, UGM, Glöckle, Elster (2002)

Hanhart, Pelaez, Rios (2008)



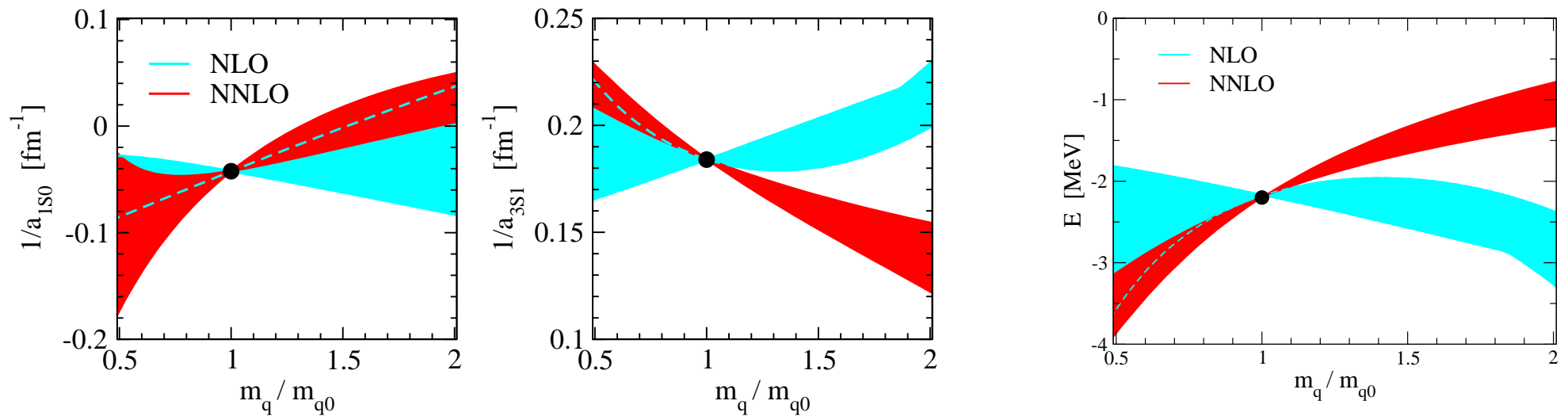
$$\frac{g^2}{t-M^2} = -\frac{g^2}{M^2} - \frac{g^2 t}{M^4} + \dots$$



# Results for the NN system

- Putting pieces together for the two-nucleon system:

$$K_{a,1S0}^q = 2.3_{-1.8}^{+1.9}, \quad K_{a,3S1}^q = 0.32_{-0.18}^{+0.17}, \quad K_{B(\text{deut})}^q = -0.86_{-0.50}^{+0.45}$$



- Extends and improves earlier work based on EFTs and models

Beane, Savage (2003), Epelbaum, UGM, Glöckle (2003), Mondejar, Soto (2007), Flambaum, Wiringa (2007), Bedaque, Luu, Platter (2011) [BLP], ...

- connection to lattice QCD results → later

# Impact on BBN

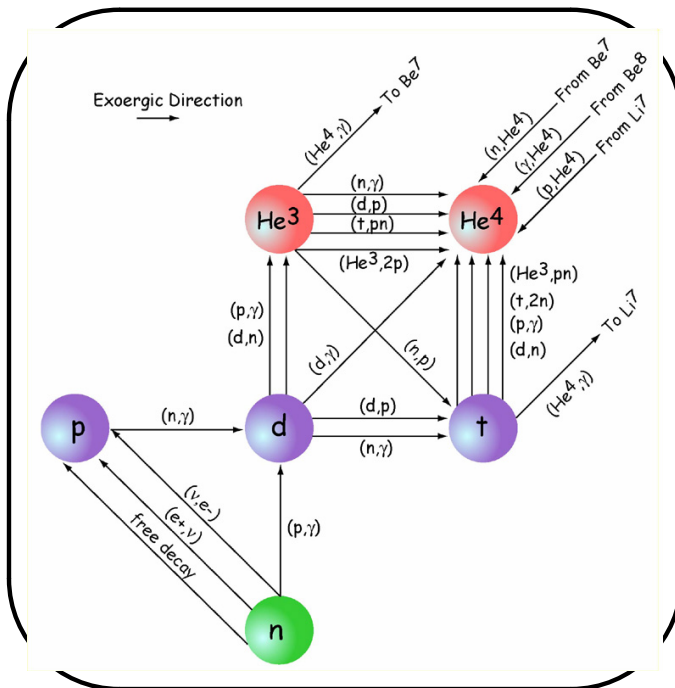
quark masses:

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,  
Phys. Rev. D **87** (2013) 085018

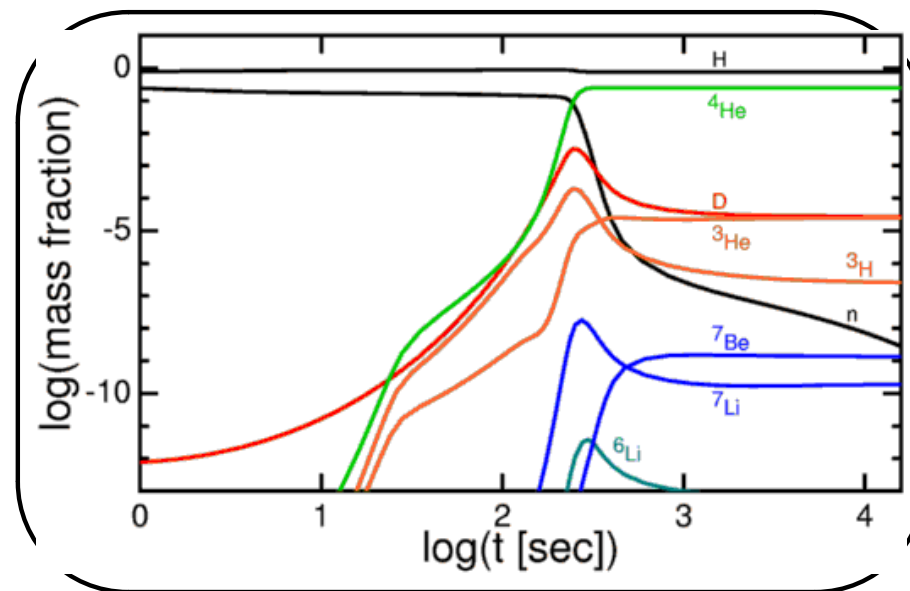
fine-structure constant:

UGM, Metsch, Meyer, [arXiv:2305.15849 [hep-th]]

# BBN network & element abundances



from Cococubed.com



from Burles, Nollett & Turner

- consider element generation up to  $^7\text{Li}$ ,  $^7\text{Be}$
- how does this network / the abundances of the elements change under variations of the quark masses/ the fine-structure constant?
- use the KAWANO code [NUC123] [Kawano, FERMILAB-PUB-92-004-A](#)

# Quark mass variations of heavier nuclei

- In BBN, we also need the variation of  ${}^3\text{He}$  and  ${}^4\text{He}$ . All other BEs are kept fixed.
- use the method of BLP: Bedaque, Luu, Platter, PRC 83 (2011) 045803

$$K_{A\text{He}}^q = K_{a, 1S0}^q K_{A\text{He}}^{a, 1S0} + K_{\text{deut}}^q K_{A\text{He}}^{\text{deut}}, \quad A = 3, 4$$

with

$$K_{3\text{He}}^{a, 1S0} = 0.12 \pm 0.01, \quad K_{3\text{He}}^{\text{deut}} = 1.41 \pm 0.01$$

$$K_{4\text{He}}^{a, 1S0} = 0.037 \pm 0.011, \quad K_{4\text{He}}^{\text{deut}} = 0.74 \pm 0.22$$

so that

$$\Rightarrow \boxed{K_{3\text{He}}^q = -0.94 \pm 0.75, \quad K_{4\text{He}}^q = -0.55 \pm 0.42}$$

$\Rightarrow$  calculate BBN response matrix of primordial abundances  $Y_a$   
at fixed baryon/photon ratio [first in the isospin limit]

# Limits for the quark mass variation

- Average of  ${}^2\text{H}$  and  ${}^4\text{He}$ :

$$\frac{\delta m_q}{m_q} = 0.02 \pm 0.04$$

- in contrast to earlier studies, we provide reliable error estimates (EFT)
- but: BLP find a stronger constraint due to the neutron life time (affects  $Y({}^4\text{He})$ )
- re-evaluate this under the model-independent assumption that  
*all* quark & lepton masses vary with the **Higgs VEV**  $v$  (CHPT w/ virtual photons)

⇒ results are dominated by the  ${}^4\text{He}$  abundance:

$$\left| \frac{\delta v}{v} \right| = \left| \frac{\delta m_q}{m_q} \right| \leq 0.9\%$$

- Presently updated: larger networks, 4 BBN codes and improved  $m_q$  variations  
UGM, Metsch, Meyer, on-going

# Fine-structure constant dependence in BBN I

UGM, Metsch, Meyer, 2305.15849 [hep-th]

- Many places where  $\alpha_{EM}$  appears directly or indirectly

↳ reaction rates, Coulomb penetration,  $\beta$ -decays,  $Q$ -values, ... note  $T$ -dep.

- New ingredients:

↳ 4 different BBN codes

PRIMAT, PArthENoPE, AlterBBN, NUC123

↳ new value  $(m_n - m_p)^{QED}$

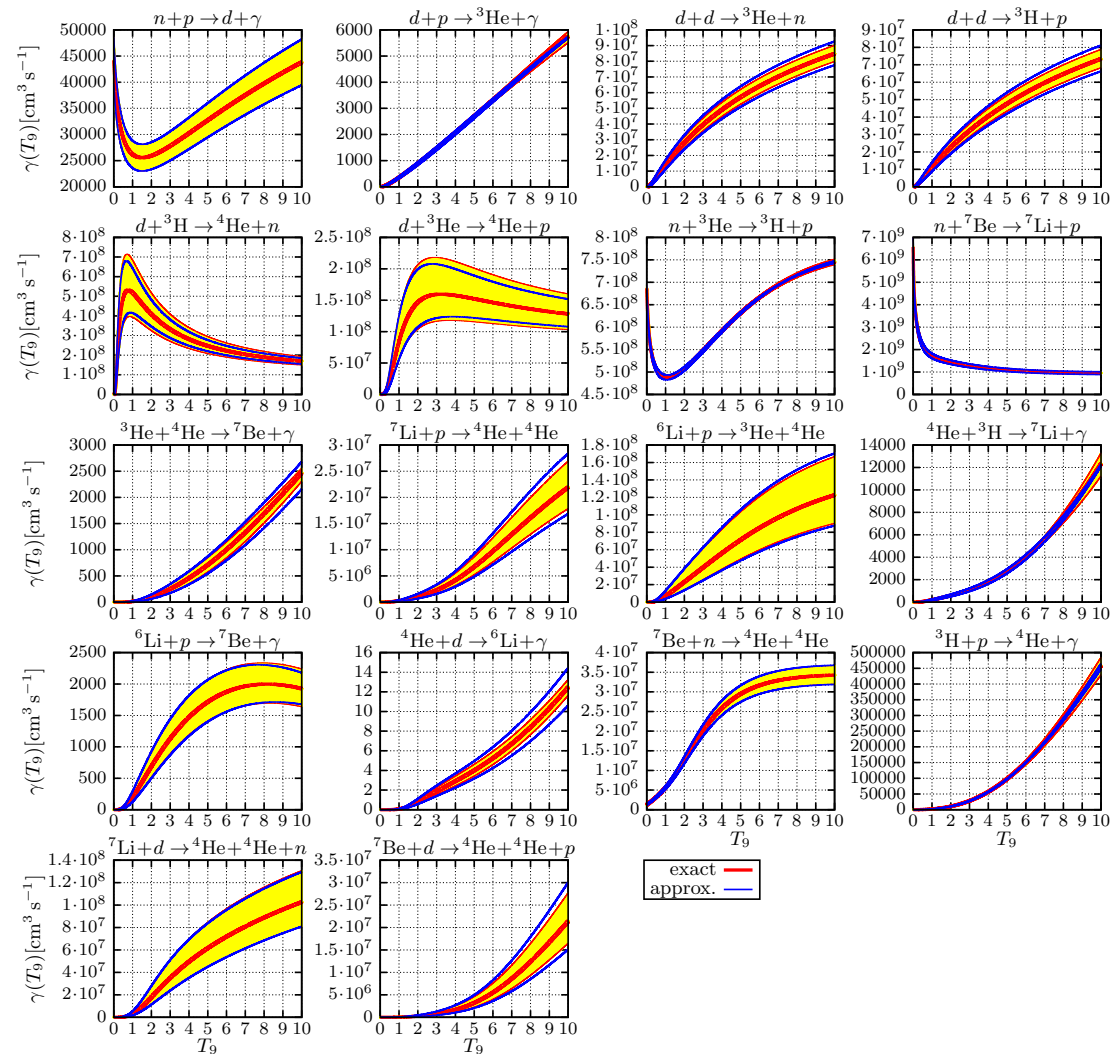
Gasser et al., Phys. Lett.B **814** (2021) 136087

↳ Coulomb energies from NLEFT

Elhatisari et al., 2210.17488 [nucl-th]

↳ modelling of reaction rates

⇒  $\alpha_{EM}$ -dependence of the rates for  $\delta\alpha_{EM} = \pm 10\%$





# Fine-structure constant dependence in BBN II

UGM, Metsch, Meyer, 2305.15849 [hep-th]

- Baryon-to-photon ratio  $\eta$  from PDG

$$\hookrightarrow \eta = 6.18 \cdot 10^{-10}$$

- Neutron lifetime  $\tau_n = 879.4$  s

- Compare w/ measured abundances

$\hookrightarrow$  from  ${}^2\text{H} + {}^4\text{He}$ :

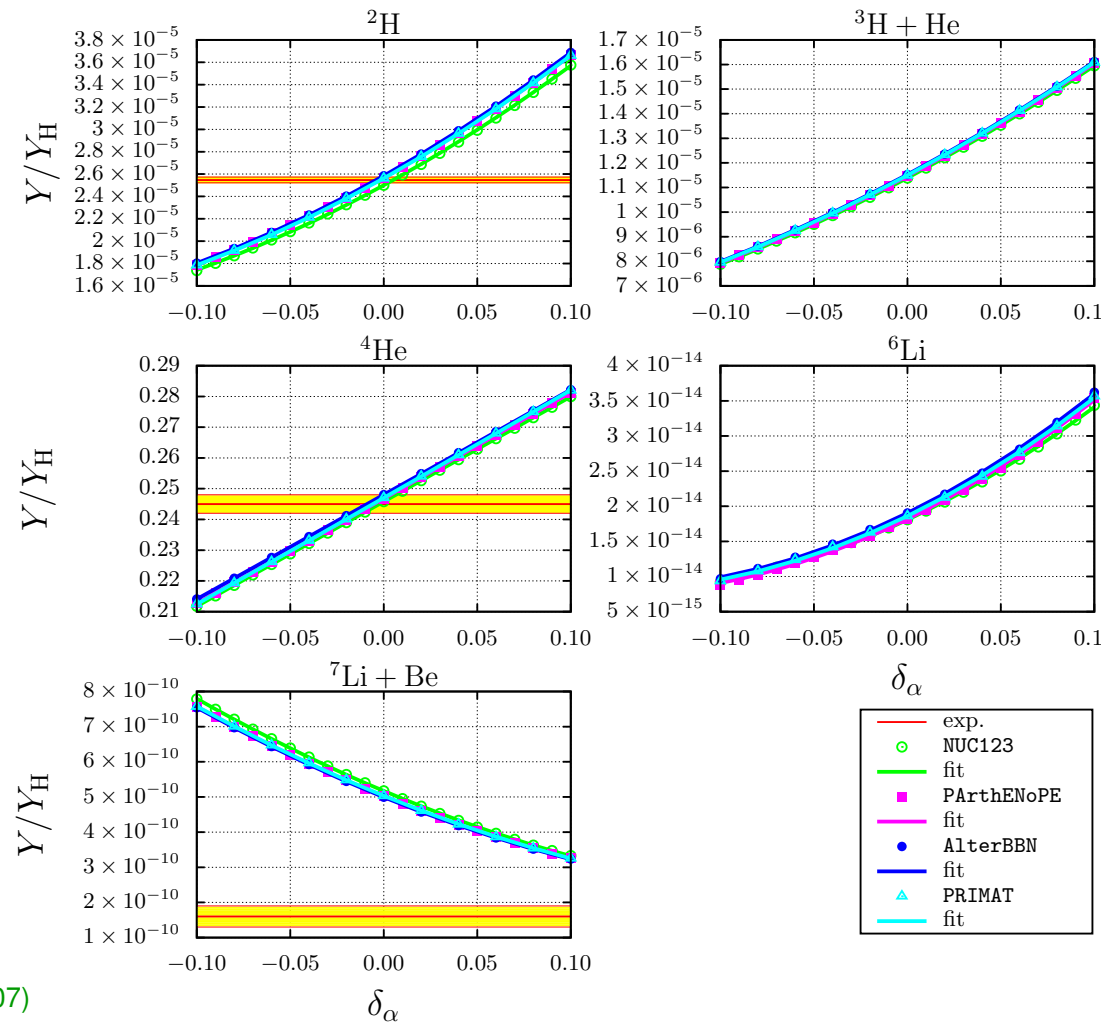
$$\left| \frac{\delta\alpha_{\text{EM}}}{\alpha_{\text{EM}}} \right| < 0.02$$

- 4 codes give consistent results

- improves on earlier works

Bergstrem et al. (1999), Nollett, Lopez (2002), Coc et al. (2007), Dent et al. (2007)

- $Y({}^7\text{Li})$  is overpredicted (“Lithium-problem”)

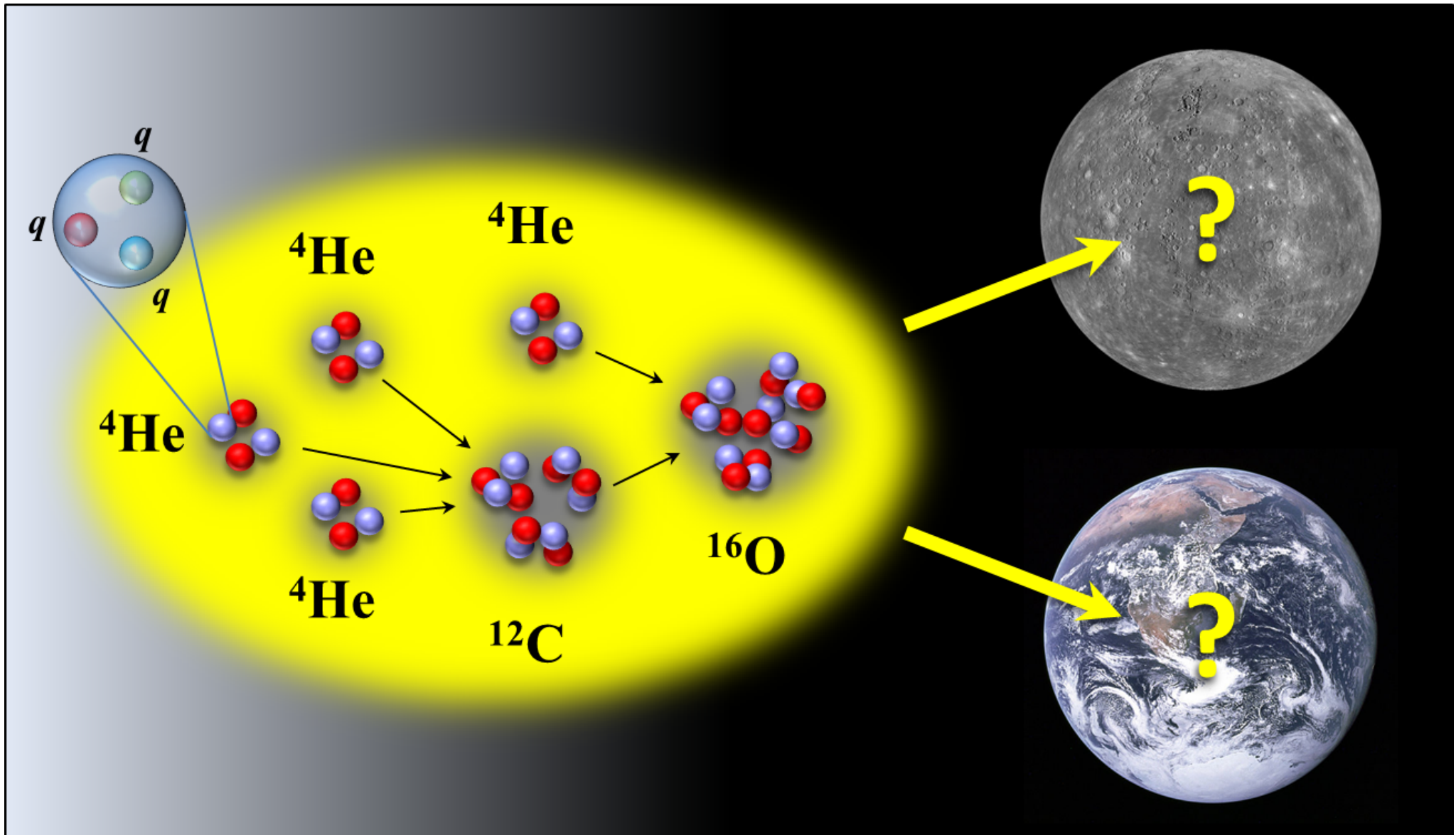


# The fate of carbon-based life as a function of the quark mass

Epelbaum, Krebs, Lähde, Lee, UGM  
Phys. Rev. Lett. **110** (2013) 112502; Eur. Phys. J. **A 48** 82 (2013)  
update: Lähde, UGM, Epelbaum, Eur. Phys. J. **A 56** (2020) 89  
review: UGM, Sci. Bull. **60** (2015) 43

# Fine-tuning of the fundamental parameters

Fig. courtesy Dean Lee

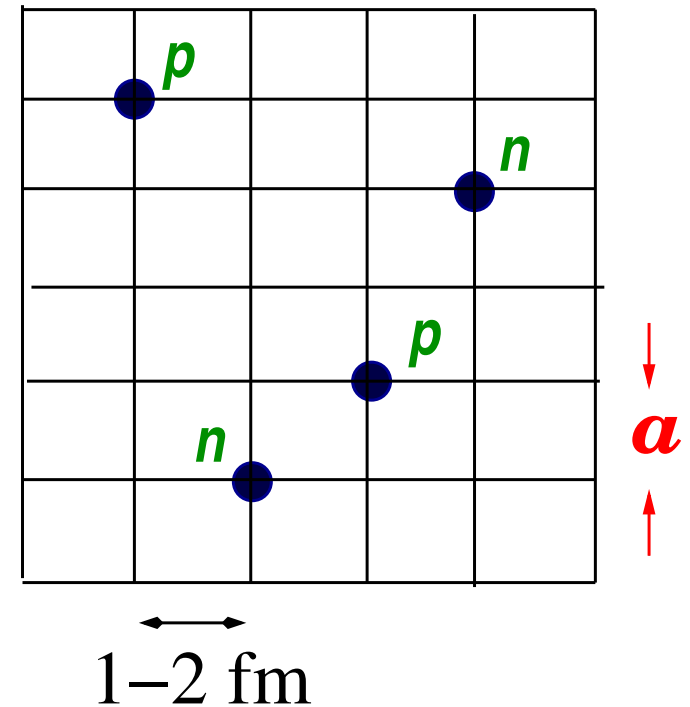


# The tool: Nuclear lattice effective field theory

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000), Lee, Schäfer (2004), . . .  
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem
- discretize space-time  $V = L_s \times L_s \times L_s \times L_t$ :  
nucleons are point-like particles on the sites
- discretized chiral potential w/ pion exchanges  
and contact interactions + Coulomb  
→ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773
- typical lattice parameters

$$p_{\max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry  
E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302
- physics independent of the lattice spacing for  $a = 1 \dots 2 \text{ fm}$

Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

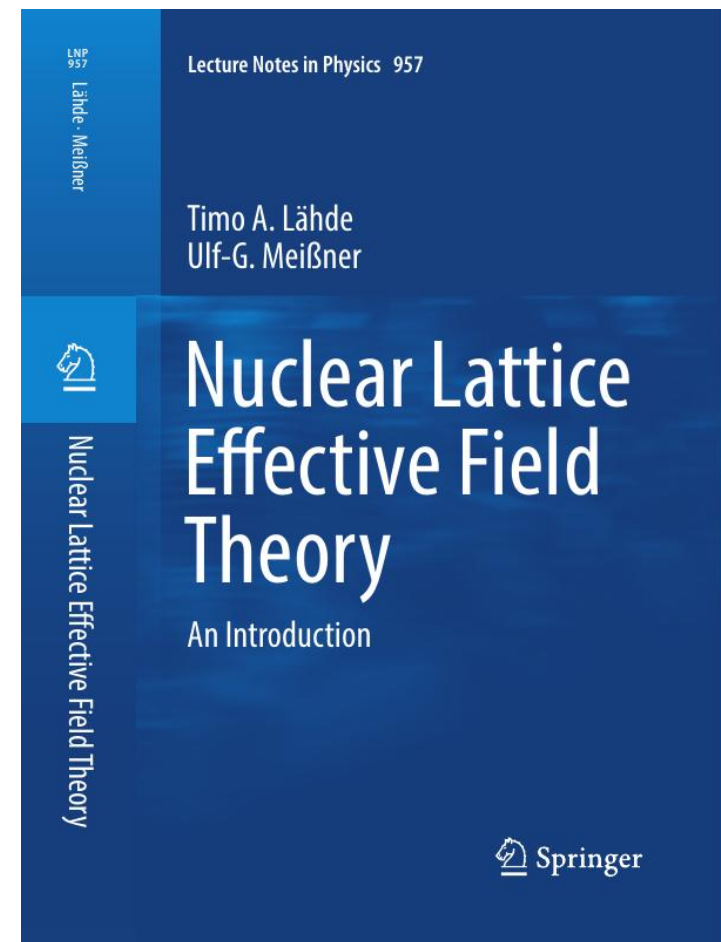
# The tool: Nuclear lattice effective field theory II

- For all details on chiral EFT on a lattice

T. Lähde & UGM

*Nuclear Lattice Effective Field Theory - An Introduction*  
Springer Lecture Notes in Physics **957** (2019) 1 - 396

- Computational equipment



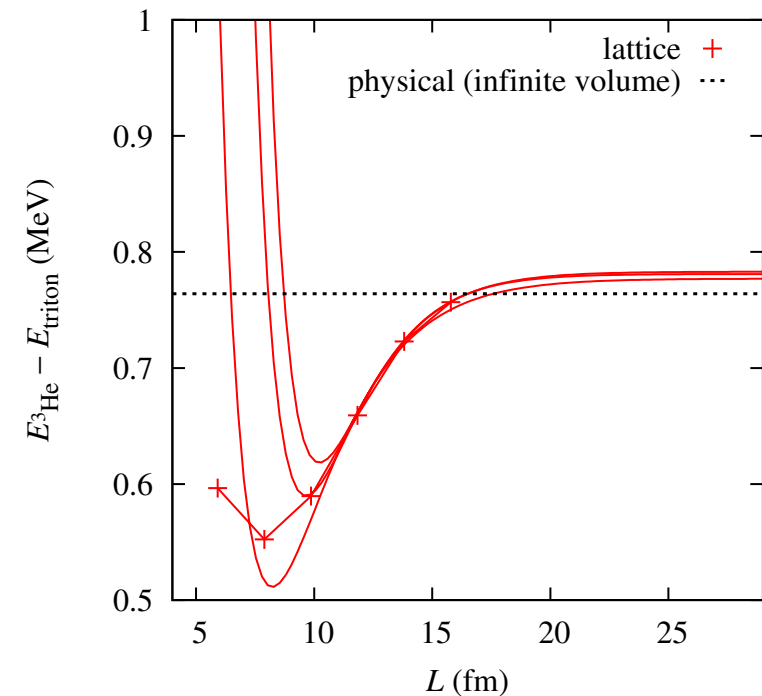
# Some early results: Validation of the method

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335

Lähde, Epelbaum, Krebs, Lee, UGM, Rupak, Phys. Lett. B **732** (2014) 110; Phys. Rev. Lett. **112** (2014) 102501

- Some groundstate energies and differences

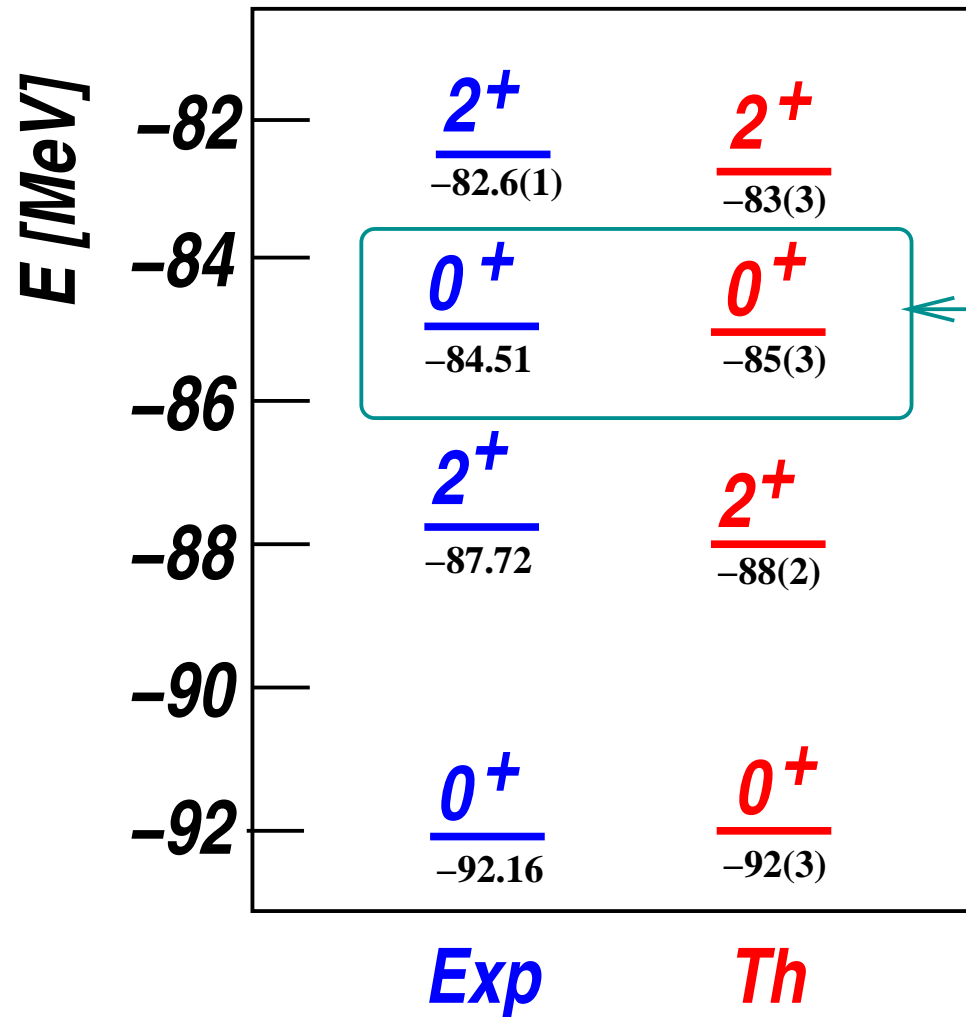
E [MeV]	NLEFT	Exp.
${}^3\text{He} - {}^3\text{H}$	0.78(5)	0.76
${}^4\text{He}$	-28.3(6)	-28.3
${}^8\text{Be}$	-55(2)	-56.5
${}^{12}\text{C}$	-92(3)	-92.2
${}^{16}\text{O}$	-131(1)	-127.6
${}^{20}\text{Ne}$	-166(1)	-160.6
${}^{24}\text{Mg}$	-198(2)	-198.3
${}^{28}\text{Si}$	-234(3)	-236.5



- promising results [much improved by now]
- excited states more difficult, but also doable

# The spectrum of carbon-12 A.D. 2011

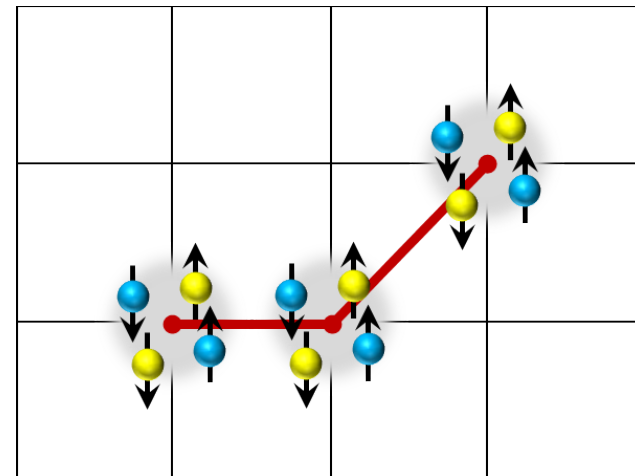
- After  $8 \cdot 10^6$  hrs JUGENE/JUQUEEN (and “some” human work)



⇒ First ab initio calculation of the Hoyle state ✓

Hoyle

Structure of the Hoyle state:



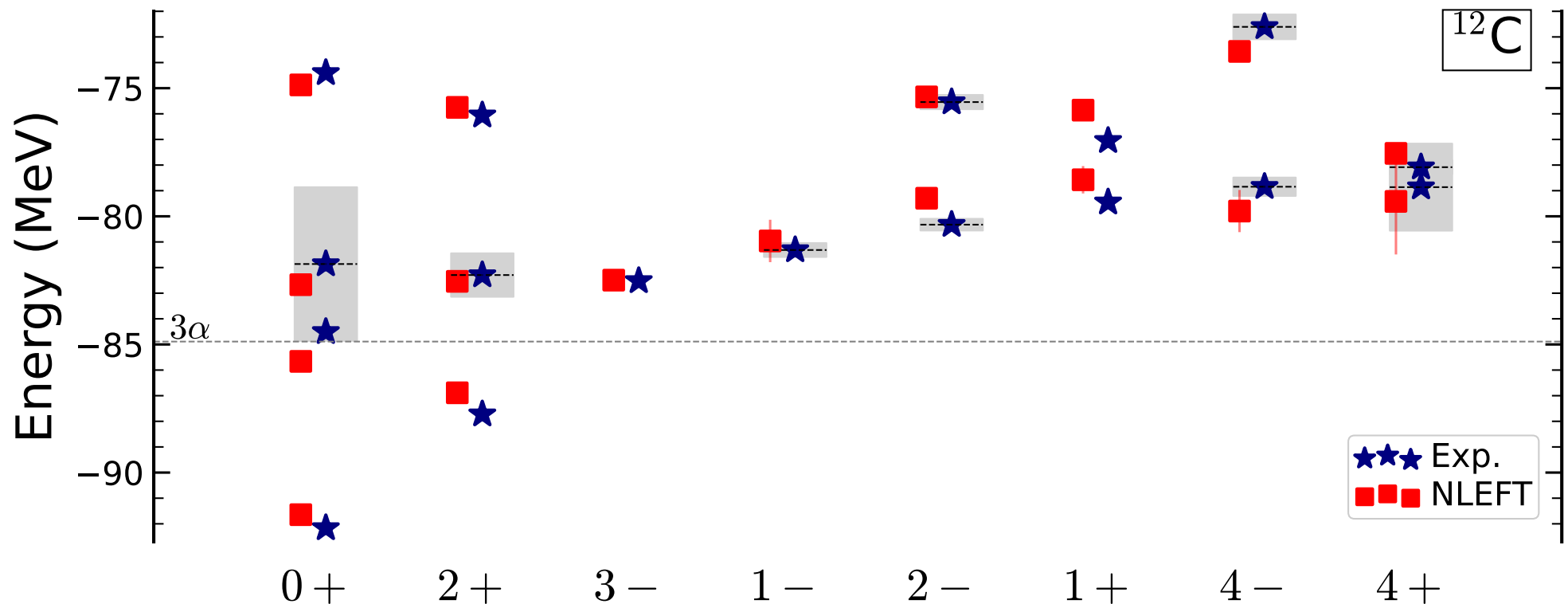
Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501

# The spectrum of carbon-12 A.D. 2023

- with much improved algorithms and methods:

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777



→ solidifies earlier NLEFT statements about the structure of the  $0_2^+$  and  $2_2^+$  states



# Pion mass dependence from MC simulations

- Consider pion mass changes as *small perturbations* for an energy (difference)  $E_i$

$$\left. \frac{\partial E_i}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = \left. \frac{\partial E_i}{\partial M_\pi^{\text{OPE}}} \right|_{M_\pi^{\text{phys}}} + x_1 \left. \frac{\partial E_i}{\partial m_N} \right|_{m_N^{\text{phys}}} + x_2 \left. \frac{\partial E_i}{\partial g_{\pi N}} \right|_{g_{\pi N}^{\text{phys}}} \\ + x_3 \left. \frac{\partial E_i}{\partial C_0} \right|_{C_0^{\text{phys}}} + x_4 \left. \frac{\partial E_i}{\partial C_I} \right|_{C_I^{\text{phys}}}$$

with

$$x_1 \equiv \left. \frac{\partial m_N}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}, \quad x_2 \equiv \left. \frac{\partial g_{\pi N}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}, \quad x_3 \equiv \left. \frac{\partial C_0}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}, \quad x_4 \equiv \left. \frac{\partial C_I}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}}$$

⇒ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the  $x_i$

- $x_1$  and  $x_2$  can be obtained from LQCD plus CHPT
- $x_3$  and  $x_4$  can be obtained from NN scattering and its  $M_\pi$ -dependence →  $\bar{A}_{s,t}$

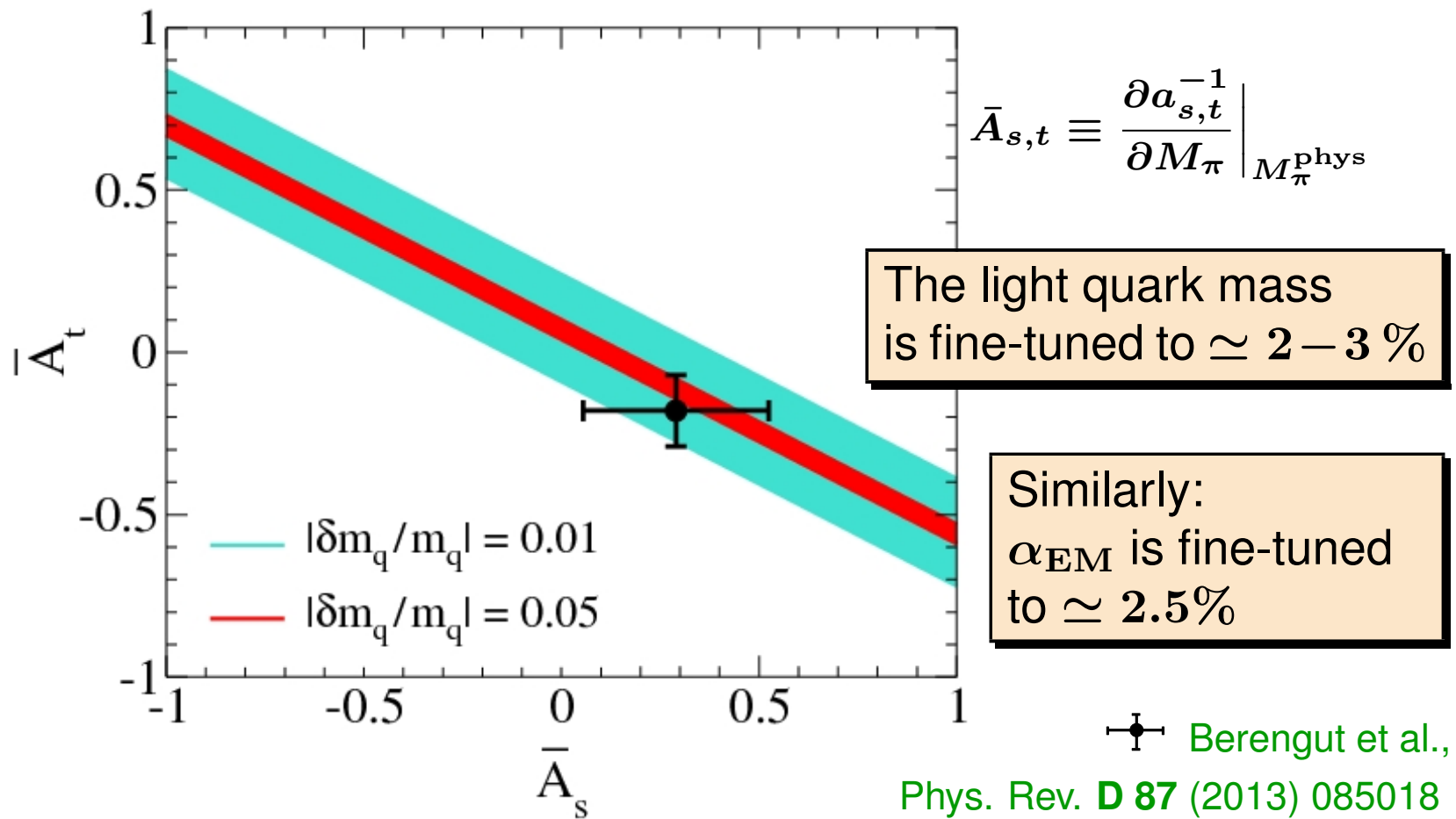


# The end-of-the-world plot I

- $|\delta(\Delta E_{h+b})| < 100 \text{ keV}$

Oberhummer et al., Science (2000)

$$\rightarrow \left| \left( 0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$



# An update on fine-tunings in the triple-alpha process <sup>36</sup>

Lähde, UGM, Epelbaum, Eur. Phys. J A 56 (2020) 89

- Use lattice data to determine  $\bar{A}_s$  and  $\bar{A}_t$ :

$$\bar{A}_s = 0.54(24) , \quad \bar{A}_t = 0.33(16)$$

↪  $\bar{A}_s$  is consistent w/ earlier determination

↪  $\bar{A}_t$  changes sign compared to earlier determination

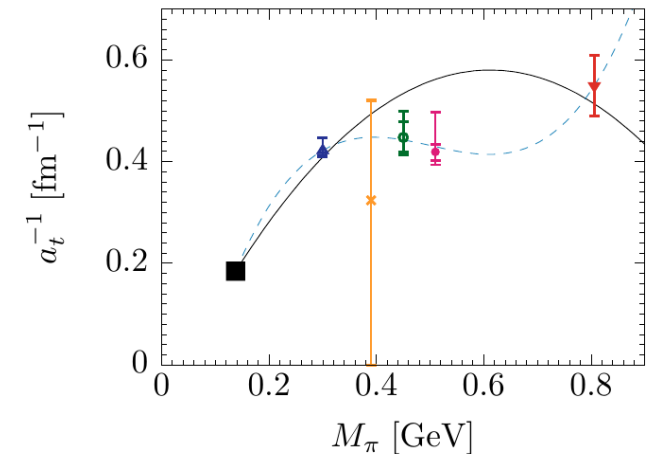
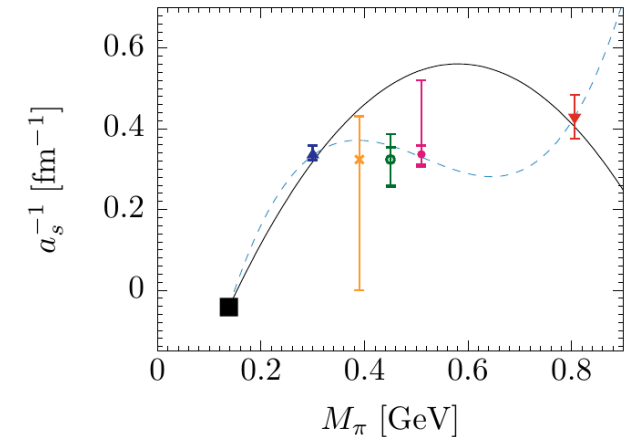
- update  $x_1$  and  $x_2$  using better LQCD data:

$$x_1 = 0.84(7) , \quad x_2 = -0.053(16)$$

↪  $x_1$  and  $x_2$  more precise

↪  $x_2$  now has a definite sign

⇒ update end-of-the-world plot



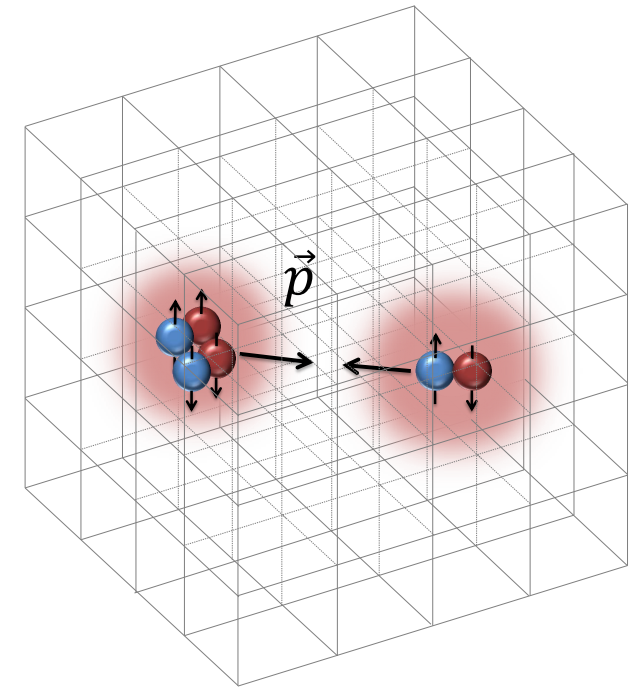
Beane et al. (2012)  
Yamazaki et al. (2015)  
Orginos et al. (2015)  
Beane et al. (2013)  
Yamazaki et al. (2012)



# Quark mass dependence of alpha-alpha scattering

# Nucleus-nucleus scattering on the lattice

- Processes involving  $\alpha$ -particles and  $\alpha$ -type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars
- Ab initio calculations of scattering and reactions using continuum methods suffer from very unfavorable computational scaling with the number of nucleons  $A$  in the clusters (either factorial or exponential in  $A$ )
- This is very different in NLEFT:



Lattice EFT computational scaling  $\Rightarrow (A_1 + A_2)^2$

Rupak, Lee, Phys. Rev. Lett. **111** (2013) 032502  
Pine, Lee, Rupak, Eur. Phys. J. A **49** (2013) 151  
Elhatisari, Lee, Phys. Rev. C **90** (2014) 064001  
Elhatisari et al., Phys. Rev. C **92** (2015) 054612  
Elhatisari, Lee, UGM, Rupak, Eur. Phys. J. A **52** (2016) 174







# Alpha-alpha scattering in the multiverse

Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001

- Now vary the light quark mass  $m_q$  and the fine-structure constant  $\alpha_{EM}$

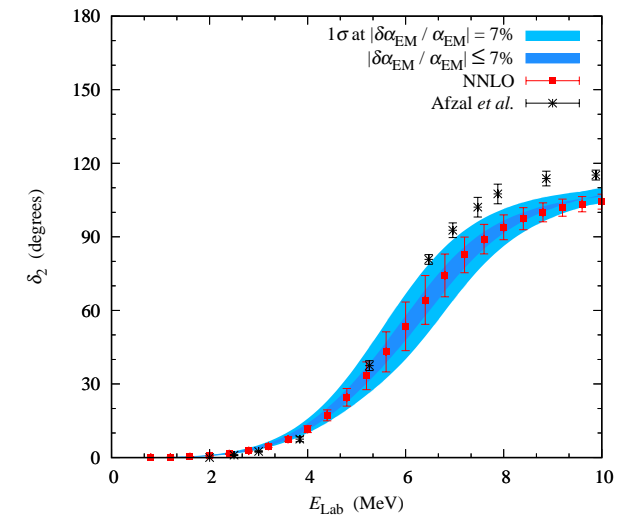
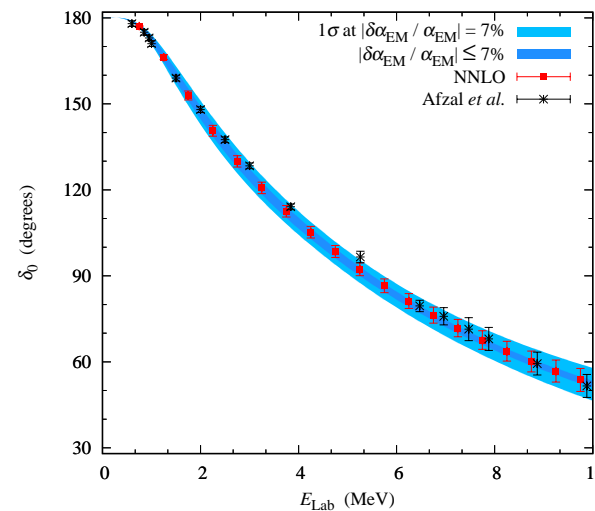
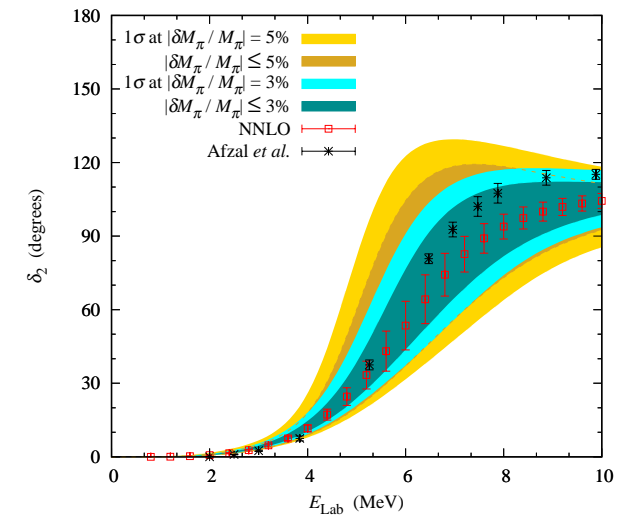
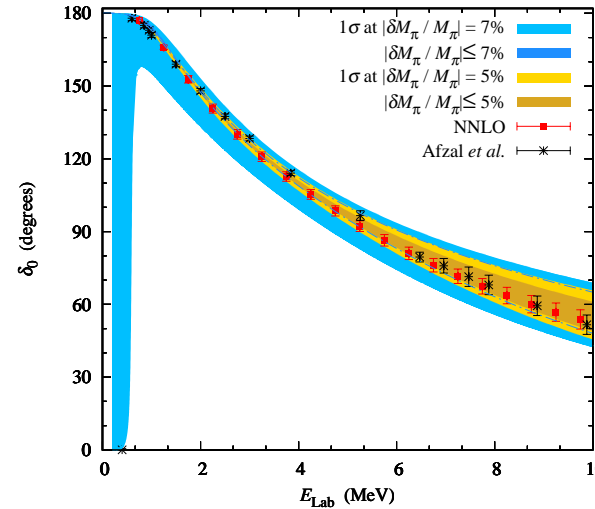
⇒ Dramatic effect in the S-wave  
for  $\delta M_\pi / M_\pi \simeq 7\%$

⇒ D-wave resonance requires  
 $\delta M_\pi / M_\pi \lesssim 3\%$

⇒ S- and D-wave phase shifts  
tolerate  $\delta\alpha_{EM} / \alpha_{EM} \lesssim 7\%$

⇒ weaker bounds as given by the  
position of the Hoyle state  
but independent of stellar modelling!

- in a next step, consider  $\alpha + {}^8\text{Be} \rightarrow {}^{12}\text{C}$   
as function of  $m_q$  and  $\alpha_{EM}$



- Chiral nuclear EFT: best approach to nuclear forces and few-body systems
- Study of the nuclear force as a function of the quark masses &  $\alpha_{\text{EM}}$ 
  - pion-exchanges straightforward, contact interactions require modeling / LQCD
- Impact on BBN:  $|\delta m_q / m_q| \leq 0.9\%$   $\hookrightarrow$  requires update
- Variations of  $\alpha_{\text{EM}}$ : many sources, new input  $\rightarrow |\delta \alpha_{\text{EM}} / \alpha_{\text{EM}}| \leq 2\%$
- Nuclear lattice simulations as a new quantum many-body approach
  - allow to vary the parameters of QCD+QED
  - investigate changes in nuclear properties + scattering can also be done
- Fine-tuning of  $m_{\text{quark}}$  and  $\alpha_{\text{EM}}$   $\rightarrow$  viability of carbon-oxygen based life
  - $\Rightarrow$  changes in  $m_q$  of about 0.5 % and in  $\alpha_{\text{EM}}$  of about 7.5% are allowed
  - $\Rightarrow$  LQCD required to reduce the uncertainties!  $\hookrightarrow$  challenge!
  - $\Rightarrow$  Sensitivity of  $\alpha$ - $\alpha$  scattering to  $m_q$  and  $\alpha_{\text{EM}}$  worked out

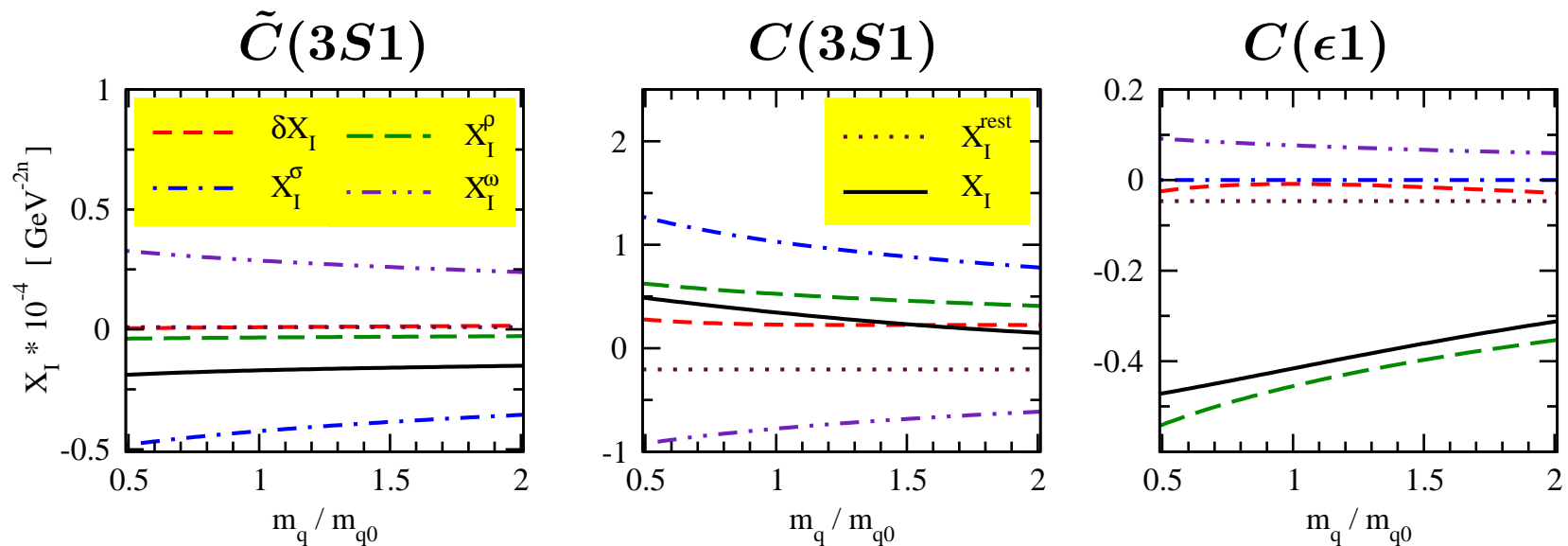
$\Rightarrow$  conditions for life are fine-tuned

# SPARES

# QUARK MASS DEP. of the SHORT-DISTANCE TERMS

- Consider a typical OBEP with  $M = \sigma, \rho, \omega, \delta, \eta$
- Quark mass dependence of the sigma and rho from unitarized CHPT  
Hanhart, Pelaez, Rios (2008)
- ⇒  $K_{M_\sigma}^q = 0.081 \pm 0.007, \quad K_{M_\rho}^q = 0.058 \pm 0.002$
- ⇒ couplings appear quark mass independent (requires refinement in the future)
- assume a) that  $K_\omega^q = K_\rho^q$  and b) neglect dep. of  $\delta, \eta$

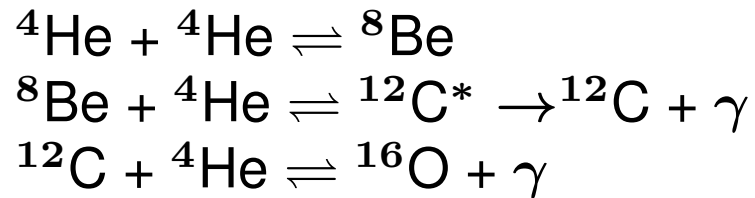
⇒



# A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: **triple- $\alpha$  process**

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, ...



- Hoyle's contribution: calculation of relative abundances of  ${}^4\text{He}$ ,  ${}^{12}\text{C}$  and  ${}^{16}\text{O}$

$\Rightarrow$  need a resonance close to the  ${}^8\text{Be} + {}^4\text{He}$  threshold at  $E_R = 0.35$  MeV

$\Rightarrow$  this corresponds to a  $J^P = 0^+$  excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at

$$E - E(\text{g.s.}) = 7.653 \pm 0.008 \text{ MeV}$$

Dunbar et al. 1953, Cook et al. 1957

- still on-going experimental activity, e.g. EM transitions at SDALINAC

M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501

- and how about theory ?  $\rightarrow$  this talk

- side remark: NOT driven by anthropic considerations

H. Kragh, Arch. Hist. Exact Sci. 64 (2010) 721







# RESULTS

- putting pieces together:

$$\left. \frac{\partial \Delta E_h}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.455(35) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.744(24) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.056(10)$$

$$\left. \frac{\partial \Delta E_b}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.117(34) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.189(24) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.012(9)$$

- $x_1$  and  $x_2$  only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

# INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$   
 $\Rightarrow \Delta E_h$  and  $\Delta E_b$  cannot be independently fine-tuned
- Within error bars,  $\partial\Delta E_h/\partial M_\pi$  &  $\partial\Delta E_b/\partial M_\pi$  appear unaffected by the choice of  $x_1$  and  $x_2 \rightarrow$  indication for  $\alpha$ -clustering

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^* - 3E_4$$

$$\left. \frac{\partial\Delta E_{h+b}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} = -0.571(14) \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} - 0.934(11) \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{phys}}} + 0.069(6)$$

$\Rightarrow$  quark mass dependence of the scattering lengths discussed earlier

