Rank 5/2 Liouville irregular block and its applications to gauge theory

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This talk is based on the paper: A note on rank 5/2 Liouville irregular block, Painlevé I and the H_0 Argyres-Douglas theory
Authors: H.P. and R.Poghossian
e-Print:

We study 4d type H_0 Argyres-Douglas theory in Ω-background by constructing Liouville irregular state of rank 5/2. The results are compared with generalized holomorphic anomaly approach, which provides order by order expansion in Ω-background parameters $\epsilon_{1,2}$. Another crucial test of our results provides comparison with respect to Painlevé I tau-functon, which was expected to be hold in self-dual case $\epsilon_1 = -\epsilon_2$.

Outline

Liouville CFTs

In any CFT the energy-momentum tensor has two nonzero components: the holomorphic ant anti-holomorphic fields $T(z)$ and $\bar{T}(\bar{z})$.

$$T(z)T(0) = \frac{c/2}{z^4} + \frac{2T(0)}{z^2} + \frac{T'(0)}{z} + \cdots \tag{1}$$

Laurent series: $T(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$
Virasoro algebra: $[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}\,.$
The central charge of Liouville CFT is

$$c = 1 + 6Q^2, \quad \text{where} \quad Q = (b + 1/b). \tag{2}$$

Primary fields are $V_\alpha = \exp 2\alpha\varphi$ with dimension

$$\Delta_\alpha = \alpha(Q - \alpha). \tag{3}$$

Primary state: $L_n|\Delta\rangle = 0$ for $n > 0$ and $L_0|\Delta\rangle = \Delta|\Delta\rangle$

The correlation function can be written as a linear combination of conformal blocks

$$\langle V_{\alpha_1}(\infty)V_{\alpha_2}(1)V_{\alpha_3}(q)V_{\alpha_4}(0)\rangle$$

The 4d $N=2$ partition function of $SU(2)$ gauge theories coincides with standard Liouville theory conformal block (AGT).
The limiting procedure which defines Argyres-Douglas theories from SU(2) gauge theories has a simple interpretation as a collision limit in Liouville theory, which produces irregular vertex operators from the collision of several standard vertex operators. Thus the Argyres-Douglas partition function correspond to irregular conformal blocks.

Rank 2 irregular state

D. Gaiotto and J. Teschner [1]

The rank 2 irregular states $|I^{(2)}\rangle$ in 2d Liouville conformal field theory, which depend on two sets of parameters $\mathbf{c} = \{c_0, c_1, c_2\}$ and $\boldsymbol{\beta} = \{\beta_0, \beta_1\}$, are defined by

$$
\begin{aligned}
L_k|I^{(2)}(c_0,c_1,c_2,\beta_0,\beta_1)\rangle &= \mathcal{L}_k|I^{(2)}(c_0,c_1,c_2,\beta_0,\beta_1)\rangle, \qquad k = 0,\dots 4 \\
L_k|I^{(2)}(c_0,c_1,c_2,\beta_0,\beta_1)\rangle &= 0, \qquad k > 4
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_0 &= c_0(Q-c_0) + c_1\partial_{c_1} + 2c_2\partial_{c_2} \\
\mathcal{L}_1 &= 2c_1(Q-c_0) + c_2\partial_{c_1} \\
\mathcal{L}_2 &= -c_1^2 + c_2(3Q-2c_0)\,; \quad \mathcal{L}_3 = -2c_1c_2\,; \quad \mathcal{L}_4 = -c_2^2
\end{aligned}
$$

[1] Irregular singularities in Liouville theory and Argyres-Douglas type gauge theories, I. arXiv:1203.1052

Rank $5/2$ irregular state

We have introduced a new type of irregular state, defined through relations

$$\begin{aligned} L_k|I^{(5/2)}(c_1,c_2,\Lambda_5,\beta_0,c_0)\rangle &= \mathcal{L}_k|I^{(5/2)}(c_1,c_2,\Lambda_5,\beta_0,c_0)\rangle, \; k=0,..,5 \\ L_k|I^{(5/2)}(c_1,c_2,\Lambda_5,\beta_0,c_0)\rangle &= 0, \qquad k>5 \end{aligned}$$

From

$$[L_n L_m]|I^{(5/2)}\rangle = -[\mathcal{L}_n \mathcal{L}_m]|I^{(5/2)}\rangle \tag{4}$$

we found

$$\begin{aligned} \mathcal{L}_0 &= c_1\frac{\partial}{\partial c_1}+2c_2\frac{\partial}{\partial c_2}+5\Lambda_5\frac{\partial}{\partial \Lambda_5} \\ \mathcal{L}_1 &= \frac{2c_1^2c_2^2}{\Lambda_5}+\frac{2c_2^3-3c_1\Lambda_5}{2c_2^2}\frac{\partial}{\partial c_1}+\frac{3\Lambda_5}{2c_2}\frac{\partial}{\partial c_2} \\ \mathcal{L}_2 &= \frac{\Lambda_5}{2c_2}\frac{\partial}{\partial c_1} \\ \mathcal{L}_3 &= -2c_1c_2\,; \quad \mathcal{L}_4=-c_2^2\,; \quad \mathcal{L}_5=-\Lambda_5 \end{aligned} \tag{5}$$

Similar to the integer rank cases we conjecture

$$|I^{(5/2)}(c_1, c_2, \Lambda_5; \beta_0, c_0)\rangle = f(c_0, c_1, c_2, \Lambda_5) \sum_{k=0}^{\infty} \Lambda_5^k |I_k^{(2)}(c_0, c_1, c_2; \beta_0, \beta_1)\rangle$$

The leading term $|I_0^{(2)}(c_0, c_1, c_2; \beta_0, \beta_1)\rangle$ is just the rank 2 irregular state, while the generalized descendants are some linear combinations of monomials (Given a partition $Y = 1^{n_1} 2^{n_2} 3^{n_3} \cdots$, by definition $L_{-Y} = \cdots L_{-3}^{n_3} L_{-2}^{n_2} L_{-1}^{n_1}$.)

$$L_{-Y} c_1^{r_1} c_2^{-r_2} \partial_{c_1}^{m_1} \partial_{c_2}^{m_2} |I_0^{(2)}(c_0, c_1, c_2; \beta_0, \beta_1)\rangle \quad (6)$$

where $n = |Y|$, $r_{1,2}$,$m_{1,2}$ are non-negative integers, subject to constraint

$$5k = n + m_1 + 2m_2 + 2r_2 - r_1 \quad (7)$$

$$|I^{(5/2)}(c_1,c_2,\Lambda_5;\beta_0,c_0)\rangle = f(c_0,c_1,c_2,\Lambda_5)\sum_{k=0}^{\infty}\Lambda_5^k|I_k^{(2)}(c_0,c_1,c_2;\beta_0,\beta_1)\rangle \quad (8)$$

where

$$f(c_0,c_1,c_2,\Lambda_5) = c_2^{\rho_2}\Lambda_5^{\rho_5}\exp(S(c_0,c_1,c_2,\Lambda_5)) \quad (9)$$

$$\begin{aligned} S(c_0,c_1,c_2,\Lambda_5) &= -\frac{2c_1^2c_2^4}{3\Lambda_5^2}+\frac{4c_1c_2^7}{27\Lambda_5^3}+\frac{4\left(c_2^4-6c_1c_2\Lambda_5\right)^{5/2}-4c_2^{10}}{405\Lambda_5^4} \\ &+ \frac{8\left(c_2^4-6c_1c_2\Lambda_5\right)^{5/4}\left(c_0-\frac{3Q}{2}\right)}{15\Lambda_5^2}+\frac{c_1^2\left(Q-c_0\right)}{c_2} \end{aligned} \quad (10)$$

$$2\rho_2+5\rho_5 = c_0(Q-c_0) \quad (11)$$

$$\rho_2 = c_0(2c_0-7Q)+\frac{71}{12}Q^2-\frac{1}{12} \quad (12)$$

Here is the result for level one descendant

$$|I_1^{(2)}(c_0,c_1,c_2;\beta_0)\rangle = \left[\frac{1}{6c_2^2}\,L_{-1} - \frac{5c_1}{6c_2^2}\partial_{c_2} + \left(\frac{c_1^2}{2c_2^3} - \frac{2a}{3c_2^2}\right)\partial_{c_1} - \frac{c_1\left(Q^2-12Qa+16a^2-1\right)}{8c_2^3} - \frac{11c_1^3(Q+2a)}{12c_2^4}\right]|I^{(2)}(c_0,c_1,c_2;\beta_0)\rangle$$

where $a = c_0 - 3Q/2$. In our paper we have also calculated the level 2 and 3 descendants.

$$|I^{(5/2)}(c_1,c_2,\Lambda_5;\beta_0,c_0)\rangle = f(c_0,c_1,c_2,\Lambda_5)\sum_{k=0}^{\infty}\Lambda_5^k|I_k^{(2)}(c_0,c_1,c_2;\beta_0,\beta_1)\rangle \quad (13)$$

The conformal block related to the partition function of the $\mathcal{H}_0$ AD theory:

$$\mathcal{Z}_{\mathcal{H}_0} = \langle 0 | I^{(5/2)}(c_1, c_2, \Lambda_5; 0, c_0) \rangle \tag{14}$$

Inserting generators $L_{0,1}$ in the vacuum amplitude $\langle 0|I^{(2)}\rangle$ we get

$$\begin{aligned} &c_0(Q - c_0) + (c_1\partial_{c_1} + 2c_2\partial_{c_2}) \log\langle 0|I^{(2)}\rangle = 0 \\ &2c_1(Q - c_0) + c_2\partial_{c_1} \log\langle 0|I^{(2)}\rangle = 0 \end{aligned} \tag{15}$$

which up to an inessential $c_{1,2}$ independent constant give

$$\langle 0|I^{(2)}\rangle = c_2^{-\frac{c_0(Q-c_0)}{2}} e^{-\frac{c_1^2(Q-c_0)}{c_2}} \tag{16}$$

$$Z_{\mathcal{H}_0} = Z_{\mathcal{H}_0,\text{tree}} Z_{\mathcal{H}_0,\text{inst}} \tag{17}$$

Here is our result

$$Z_{\mathcal{H}_0\text{tree}} = c_2^{-\frac{c_0(Q-c_0)}{2}+\rho_2} \Lambda_5^{\rho_5} e^{-\frac{c_1^2(Q-c_0)}{c_2}+S} \tag{18}$$

$$Z_{\mathcal{H}_0\text{inst}} = 1 + \frac{c_1}{8c_2^3} \left(1 - 71Q^2 + 30c_0(3Q - c_0)\right) \Lambda_5 + \ldots \tag{19}$$

For the normalized expectation value of the stress tensor we have

$$\phi_2(z) \;=\; -\frac{\langle 0|T(z)|I^{(5/2)}\rangle}{\langle 0||I^{(5/2)}\rangle} = \frac{2v}{z^4} + \frac{2c_1c_2}{z^5} + \frac{c_2^2}{z^6} + \frac{\Lambda_5}{z^7} \tag{20}$$

with

$$v = -\frac{\Lambda_5}{4c_2}\,\partial_{c_1}\log\mathcal{Z}_{\mathcal{H}_0} \tag{21}$$

So we found

$$v = c_0c_2 + \frac{c_1^2}{2} + \left(\frac{c_1^3}{2c_2^3} - \frac{3c_0c_1}{2c_2^2}\right)\Lambda_5 + \dots \tag{22}$$

CFT gauge theory map:

$$Q = \frac{s}{\sqrt{p}};\; c_0 = \frac{a + \frac{3s}{2}}{\sqrt{p}};\; v = \frac{\hat{v}}{p};\; \phi_2 = \frac{\hat{\phi}_2}{p};\; \Lambda_5 = \frac{\hat{\Lambda}_5}{p};\; c_i = \frac{\hat{c}_i}{\sqrt{p}};\; i = 1, 2$$

where $s = \epsilon_1 + \epsilon_2$ and $p = \epsilon_1\epsilon_2$.

$$\hat{\phi}_2(z) = \frac{2\hat{v}}{z^4} + \frac{2\hat{c}_1\hat{c}_2}{z^5} + \frac{\hat{c}_2^2}{z^6} + \frac{\hat{\Lambda}_5}{z^7} = \frac{1}{z^8}\left(\hat{\Lambda}_5 z + \hat{c}_2^2 z^2 + 2\hat{c}_1\hat{c}_2 z^3 + 2\hat{v} z^4\right) \quad (23)$$

The 1-form

$$\lambda_{SW} = \sqrt{\hat{\phi}_2(z)}\, dz \quad (24)$$

is the Saiberg-Witten differential. The period integrals along A and B-cycles can be evaluated exactly in terms of hypergeometric function, but for now notice, that A-cycle shrinks to the point $z = 0$ in $\Lambda_5 \to 0$ limit, so that one can simply expand $\sqrt{\hat{\phi}_2}$ in powers of $\hat{\Lambda}_5$ and then take the residues at $z = 0$. Here is the result up to order $O(\hat{\Lambda}_5^2)$:

$$a = \frac{1}{2\pi i}\oint_{z=0} \sqrt{\hat{\phi}_2} dz = \frac{\hat{v}}{\hat{c}_2} - \frac{\hat{c}_1^2}{2\hat{c}_2} + \left(\frac{3\hat{c}_1\hat{v}}{2\hat{c}_2^4} - \frac{5\hat{c}_1^3}{4\hat{c}_2^4}\right)\hat{\Lambda}_5 + \dots \quad (25)$$

$$\hat{v} = a\hat{c}_2 + \frac{\hat{c}_1^2}{2} + \left(\frac{\hat{c}_1^3}{2\hat{c}_2^3} - \frac{3a\hat{c}_1}{2\hat{c}_2^2}\right)\hat{\Lambda}_5 + \dots \quad (26)$$

The SW differential

$$\lambda_{SW} = \sqrt{\hat{\phi}_2}\,\frac{dz}{2\pi i} \quad \text{where} \quad \phi_2(z) = \frac{2\hat{v}}{z^4} + \frac{2c_1c_2}{z^5} + \frac{c_2^2}{z^6} + \frac{\Lambda_5}{z^7} \tag{27}$$

After performing $z = -\frac{3\hat{\Lambda}_5}{3x+\hat{c}_2^2}$ for the SW differential we get

$$\lambda_{SW} = \frac{1}{2\hat{\Lambda}_5^2}\sqrt{-4x^3 + g_2x + g_3}\,\frac{dx}{2\pi i} \tag{28}$$

with Weierstrass parameters

$$g_2 = \frac{4c_2^4}{3} - 8c_1c_2\Lambda_5\,; \quad g_3 = -\frac{8}{3}c_1c_2^3\Lambda_5 + \frac{8c_2^6}{27} + 8\Lambda_5^2\hat{v} \tag{29}$$

For the holomorphic differential we get

$$\partial_{\hat{v}}\lambda_{SW} = \frac{2}{\sqrt{-4x^3 + g_2x + g_3}}\,\frac{dx}{2\pi i} \tag{30}$$

The periods of this holomorphic differential are

$$\partial_v a = \left(\frac{3g_2}{4}\right)^{-\frac{1}{4}} {}_2F_1\left(\frac{1}{6},\frac{5}{6};1;\frac{1}{2}-\frac{1}{2}\sqrt{\frac{27g_3^2}{g_2^3}}\right) \tag{31}$$

$$\partial_v a_D = i\left(\frac{3g_2}{4}\right)^{-\frac{1}{4}} {}_2F_1\left(\frac{1}{6},\frac{5}{6};1;\frac{1}{2}+\frac{1}{2}\sqrt{\frac{27g_3^2}{g_2^3}}\right) \tag{32}$$

Above expressions can be easily integrated over v

$$a = -\frac{1}{27\Lambda_5^2}\left(\frac{3g_2}{4}\right)^{\frac{5}{4}}\left(1-\sqrt{\frac{27g_3^2}{g_2^3}}\right) {}_2F_1\left(\frac{1}{6},\frac{5}{6};2;\frac{1}{2}-\frac{1}{2}\sqrt{\frac{27g_3^2}{g_2^3}}\right) \tag{33}$$

$$a_D = \frac{i}{27\Lambda_5^2}\left(\frac{3g_2}{4}\right)^{\frac{5}{4}}\left(1+\sqrt{\frac{27g_3^2}{g_2^3}}\right) {}_2F_1\left(\frac{1}{6},\frac{5}{6};2;\frac{1}{2}+\frac{1}{2}\sqrt{\frac{27g_3^2}{g_2^3}}\right) \tag{34}$$

$$a_D = \frac{i}{2\pi}\,\partial_a\mathcal{F}_0\,; \qquad \hat{v} = -\frac{\Lambda_5}{4c_2}\partial_{\hat{c}_1}\mathcal{F}_0\,; \qquad -2\log q = \mathcal{F}_0''(a) \tag{35}$$

The holomorphic anomaly recursion

For the prepotential we have

$$\mathcal{F} = \epsilon_1\epsilon_2 \log Z = \sum_{n=0,m=0}^{\infty} (\epsilon_1+\epsilon_2)^{2n} (\epsilon_1\epsilon_2)^m F^{(n,m)} = \sum_{g=0}^{\infty} (\epsilon_1\epsilon_2)^g \mathcal{F}_g \quad (36)$$

where

$$\mathcal{F}_g = \sum_{n+m=g} \left(\frac{s^2}{p}\right)^n F^{(n,m)} \quad (37)$$

We parameterized the Ω-background (ϵ_1, ϵ_2) with the variables

$$s = \epsilon_1 + \epsilon_2 \qquad , \qquad p = \epsilon_1\epsilon_2 \quad (38)$$

There is a powerful method to compute corrections in Ω-background parameters $\epsilon_{1,2}$ based on holomorphic anomaly recursion M. Bershadsky, S. Cecotti, H. Ooguri, C. Vafa, M.-x. Huang A. Klemm, D. Krefl and J. Walcher

Consider any SW theory governed by an elliptic curve. Suppose this elliptic curve is cast in Weierstrass canonical form

$$y^2 = 4z^3 - g_2 z - g_3 \tag{39}$$

Periods of the Weierstrass elliptic curve are given by

$$\omega_i = \oint_{\gamma_i} dz/(i\pi y) \tag{40}$$

As usual the infrared coupling τ_{IR} is identified with torus parameter $\tau_{IR} = \frac{\omega_2}{\omega_1}$. It is convenient to introduce the nome given by $q = e^{\pi \mathrm{i} \tau_{IR}}$. Due to standard formulae of elliptic geometry

$$g_2 = \frac{4}{3\omega_1^4} E_4(q)\,; \qquad g_3 = \frac{8}{27\omega_1^6} E_6(q)\,; \tag{41}$$

$$E_k(q) = 1 + \frac{2}{\zeta(1-k)} \sum_{n=1}^{\infty} \frac{n^{k-1} q^{2n}}{1-q^{2n}} \qquad k = 2, 4, 6 \dots \tag{42}$$

The "flat" coordinate a and the SW prepotential $\mathcal{F}(a)$ are introduced by

$$-2\log q = \mathcal{F}_0''(a) \qquad , \qquad \omega_1(q,u) = \frac{da}{du} \tag{43}$$

$$\partial_X \mathcal{F}_g = \frac{3}{16}\left[\frac{d^2\mathcal{F}_{g-1}}{d\hat{v}^2} + \frac{p_1}{2}\frac{d\mathcal{F}_{g-1}}{d\hat{v}} + \sum_{g'=1}^{g-1}\frac{d\mathcal{F}_{g'}}{d\hat{v}}\frac{d\mathcal{F}_{g-g'}}{d\hat{v}}\right] \qquad (44)$$

using as the starting point $g = 1$ expression

$$\mathcal{F}_1 = \frac{s^2-2p}{24p}\log\Delta(\hat{v}) + \frac{1}{4}\log\frac{9g_3E_4}{2g_2E_6}\,; \qquad \Delta(\hat{v}) = g_2^3 - 27g_3^2$$

The following quantities are introduced

$$S = \frac{2}{9\omega_1(q,\hat{v})^2} = \frac{g_3(\hat{v})E_4(q)}{g_2(\hat{v})E_6(q)}\,; \quad X = SE_2(q)\,; \quad p_1 = \frac{d}{d\hat{v}}\ln S \qquad (45)$$

$\mathcal{F}_g$ is a polynomial in X of maximal degree $3(g-1)$ with rational in $\hat{v}$ coefficients.More precisely the denominators of this coefficients are equal to $\Delta(\hat{v})^{2g-2}$ and numerators are polynomials in $\hat{v}$ of maximal degree $2d_\Delta(g-1)-1$, where d_Δ is the degree of discriminant in $\hat{v}$. The gap conditions reads

$$\mathcal{F}_g \underset{\hat{v}\to\hat{v}^*}{\approx} (2g-3)!\sum_{k=0}^{g}\hat{B}_{2k}\hat{B}_{2g-2k}\frac{\epsilon_1^{2g-2k}\epsilon_2^{2k}}{a^{2g-2}} + O(a^0), \quad g>0 \qquad (46)$$

$$\hat{B}_m = \left(2^{1-m}-1\right)\frac{B_m}{m!} \qquad (47)$$

Using the holomorphic recursive algorithm for first four terms we got

$$-\tfrac{1}{2}\frac{\partial^2 \mathcal{F}_0}{\partial a^2} = \log q$$

$$\mathcal{F}_1 = \frac{s^2-2p}{24p}\log\left(g_2^3-27g_3^2\right)+\frac{1}{4}\log\frac{9g_3E_4}{2g_2E_6}$$

$$\mathcal{F}_2 = \frac{\Lambda_5^4 g_2^2 g_3}{\left(g_2^3-27g_3^2\right)^2 p^2}\left(\frac{15p^2E_2^3}{4E_6}+\frac{9p\left(11p-2s^2\right)E_2^2}{4E_4}+\frac{9\left(11p^2-12ps^2+s^4\right)E_6E_2}{4E_4^2}\right.$$
$$\left.+\frac{9p\left(7p-6s^2\right)E_4E_2}{2E_6}+\frac{3}{20}\left(299p^2-618ps^2+237s^4\right)\right)$$

$$\mathcal{F} = \sum_{g=0}^{\infty}\left(\epsilon_1\epsilon_2\right)^g\mathcal{F}_g \qquad (48)$$

Penlevé I

The equation Penlevé I

$$q_{tt} = 6q^2 + t \qquad (49)$$

one of six second order ordinary differential equations in classification scheme developed in classical works. The equation (49) can be represented in Hamiltonian form with Hamiltonian

$$\sigma(t) = \frac{q_t^2}{2} - 2q^3 - qt \qquad (50)$$

I which due to (49) itsef satisfies the equation

$$\sigma_{tt}^2 = 2(\sigma - t\sigma_t) - 4\sigma_t^3 \qquad (51)$$

τ-function of P1 is introduced through the relation

$$\tau(t) = \frac{\sigma_t}{\sigma} \qquad (52)$$

$$\tau(t) = \frac{\sigma_t}{\sigma} \tag{53}$$

According to the conjecture proposed in G. Bonelli, O. Lisovyy, K. Maruyoshi, A. Sciarappa, A. Tanzini [2] along the 5 rays in complex t-plane $\arg t = \pi, \pm 3\pi/5, \pm\pi/5$ the function $\tau(t)$ admits the following series representation

$$\begin{aligned}
\tau(t) &= s^{-\frac{1}{10}} \sum_{n\in\mathbb{Z}} e^{in\rho} \mathcal{G}(\nu+n, s)\,; \qquad 24t^5 + s^4 = 0\,, \quad s \in \mathbb{R}_{\geq 0} \\
\mathcal{G}(\nu, s) &= C(\nu, s)\left[1 + \sum_{k=1}^{\infty} \frac{D_k(\nu)}{s^k}\right] \\
C(\nu, s) &= (2\pi)^{\frac{\nu}{2}} e^{\frac{s^2}{45} + \frac{4}{5} i\nu s - \frac{i\pi\nu^2}{4}} s^{\frac{1}{12} - \frac{\nu^2}{2}} 48^{-\frac{\nu^2}{2}} G(1+\nu)
\end{aligned} \tag{54}$$

where $G(1+\nu)$ is Barnes G-function and the parameters ν, ρ are related to Stokes multipliers.

[2] On Painlevé/gauge theory correspondence, arXiv:1612.06235

The first three coefficients $D_k(\nu)$ explicitly read

$$D_1(\nu) = -\frac{i\nu(94\nu^2+17)}{96}$$
$$D_2(\nu) = -\frac{44180\nu^6+170320\nu^4+74985\nu^2+1344}{92160}$$
$$D_3(\nu) = -\frac{i\nu(4152920\nu^8+45777060\nu^6+156847302\nu^4+124622833\nu^2+13059000)}{26542080}$$

In analogy with previously known cases, it was anticipated that $\mathcal{G}(\nu, s)$ should be closely related to partition function of $\mathcal{H}_0$ theory in Ω-background with $\epsilon_1 = -\epsilon_2$. Explicitly, under identification

$$s = \frac{2\left(c_2^4 - 6c_1c_2\right)^{5/4}}{3\Lambda_5^2}\,; \qquad \nu = -ia \tag{55}$$

SUMMARY

- We have derived the rank $5/2$ conformal block
- We have derived the H_0 prepotential
- We made PI tau-functon and H_0 prepotential connection explicit.

We also discuss Nekrasov-Shatashvili limit $\epsilon_1 = 0$, accessible either by means of deformed Seiberg-Witten curve, or WKB methods.

THANKS