# $\mathcal{N} = 2$ superconformal higher spins from Harmonic superspace

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Summary and outlook

# Supersymmetry and higher spins

- Supersymmetric higher-spin theories provide a bridge between superstring theory and low-energy (super)gauge theories.

- Free massless bosonic and fermionic higher spin field theories: Fronsdal, 1978; Fang, Fronsdal, 1978.

- The natural tools to deal with supersymmetric theories are off-shell superfield methods: supersymmetry is closed on the off-shell supermultiplets with the correct sets of the auxiliary fields and so is manifest. Unconstrained superfield formulations are most preferable.

- The component approach to $4D$, $\mathcal{N} = 1$ supersymmetric free massless higher spin models: Courtright, 1979; Vasiliev, 1980.

- The complete off-shell Lagrangian formulation of $4D$ free higher spin $\mathcal{N} = 1$ models through $\mathcal{N} = 1$ superfields: Kuzenko et al, 1993, 1994.

- For long time, an off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown even for free theories.

- An off-shell manifestly $\mathcal{N} = 2$ supersymmetric unconstrained formulation of $4D, \mathcal{N} = 2$ superextension of the Fronsdal theory for integer spins was constructed, based on the harmonic superspace approach, in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 12 (2021) 016, arXiv: 2109.07639 [hep-th]..

- Manifestly $\mathcal{N} = 2$ supersymmetric off-shell cubic couplings of $4D, \mathcal{N} = 2$ to the hypermultiplets: I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 05 (2022) 104, arXiv:2202.08196 [hep-th]; JHEP **03** (2023) 036, arXiv:2212.14114 [hep-th].

- These papers opened a new area of applications of the harmonic superspace formalism, that time in $\mathcal{N} = 2$ higher-spin theories.

# Harmonic superspace

- In $4D$, the self-consistent off-shell superfield formalism for $\mathcal{N}=2$ and $\mathcal{N}=3$ theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, CQG 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z=(x^m\,,\ \theta^\alpha_i\,,\ \bar\theta^{\dot\alpha j}, u^{\pm i}), \quad u^{\pm i}\in SU(2)/U(1),\ u^{+i}u^-_i=1.$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A=(x^m_A,\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm i}),\ \theta^{+\alpha,\dot\alpha}:=\theta^{\alpha,\dot\alpha i}u^+_i,\ x^m_A:=x^m-2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_iu^+_j$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}:\quad V^{++}(\zeta_A)\,,\ \ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A)\,,\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}:\quad H^{++m}(\zeta_A)\,,\ H^{++\alpha+}(\zeta_A)\,,\ H^{++5}(\zeta_A)\,,\hat\alpha=(\alpha,\dot\alpha)$$

# $\mathcal{N} = 2$ spin 1 multiplet

- Abelian $\mathcal{N} = 2$ gauge theory,

$$V^{++}(\zeta_A)\,, \quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\; D^{++} = \partial^{++} - 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\,.$$

- Wess-Zumino gauge:

$$V^{++}(\zeta_A) = (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i u^-_k\,.$$

- $4D$ fields $\phi$, $\bar{\phi}$, $A_{\alpha\dot{\alpha}}$, $\psi^i_\alpha$, $\bar{\psi}^i_{\dot{\alpha}}$, $D^{(ik)}$ Abelian gauge $\mathcal{N} = 2$ off-shell multiplet ($8 + 8$ off-shell degrees of freedom).

- Invariant action:

$$S \sim \int d^{12}Z\,\left(V^{++}V^{--}\right),\; D^{++}V^{--} - D^{--}V^{++} = 0\,,\; \delta V^{--} = D^{--}\Lambda\,,$$
$$[D^{++}, D^{--}] = D^0, \quad D^0\,V^{\pm\pm} = \pm 2\,V^{\pm\pm}\,.$$

# $\mathcal{N} = 2$ spin 2: linearized $\mathcal{N} = 2$ supergravity

- Analogs of $V^{++}(\zeta_A)$ are the following set of analytic gauge potentials:

$$\Big(h^{++m}(\zeta_A)\,,\; h^{++5}(\zeta_A)\,,\; h^{++\hat{\mu}+}(\zeta_A)\Big), \quad \hat{\mu} = (\mu\,,\dot{\mu})\,,$$

$$\delta_\lambda h^{++m} = D^{++}\lambda^m + 2i\big(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big)\,,$$

$$\delta_\lambda h^{++5} = D^{++}\lambda^5 - 2i\big(\lambda^{+\alpha}\theta^+_\alpha - \bar{\theta}^+_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big), \delta_\lambda h^{++\hat{\mu}+} = D^{++}\lambda^{+\hat{\mu}}\,.$$

- Wess-Zumino gauge:

$$h^{++m} = -2i\theta^+\sigma^a\bar{\theta}^+\Phi^m_a + \big[(\bar{\theta}^+)^2\theta^+\psi^{m\,i}u^-_i + c.c.\big] + \ldots$$

$$h^{++5} = -2i\theta^+\sigma^a\bar{\theta}^+ C_a + \ldots\,, \quad h^{++\mu+} = \ldots$$

- The residual gauge freedom:

$$\lambda^m \;\Rightarrow\; a^m(x)\,,\; \lambda^5 \;\Rightarrow\; b(x)\,,\; \lambda^{\mu+} \;\Rightarrow\; \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu}l_{(\nu}{}^{\mu)}(x)\,.$$

- The physical fields are $\Phi^m_a, \psi^{m\,i}_\mu, C_a$ $((\mathbf{2}, \mathbf{3/2}, \mathbf{3/2}, \mathbf{1})$ on shell). The antisymmetric part of $\Phi^m_a$ is gauged away by the local "Lorentz" parameters $l_{(\nu}{}^{\mu)}(x), l_{(\dot{\nu}}{}^{\dot{\mu})}(x)$:

$$\Phi^m_a \sim \Phi_{\beta\dot{\beta}\alpha\dot{\alpha}} \Rightarrow \Phi_{(\beta\alpha)(\dot{\beta}\dot{\alpha})} + \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi\,.$$

- The invariant action:

$$S \sim \int d^4xd^8\theta du\Big(G^{++\alpha\dot\alpha}G^{--}_{\alpha\dot\alpha} + G^{++5}G^{--5}\Big),$$

$$G^{++\mu\dot\mu} := h^{++\mu\dot\mu} + 2i\big(h^{++\mu+}\bar\theta^{-\dot\mu} + \theta^{-\mu}h^{++\dot\mu+}\big)\,,$$

$$G^{++5} := h^{++5} - 2i\big(h^{++\mu+}\theta^{-}_{\mu} - \bar\theta^{-}_{\dot\mu}h^{++\dot\mu+}\big)\,,$$

$$D^{++}G^{--\mu\dot\mu} = D^{--}G^{++\mu\dot\mu}\,,\quad D^{++}G^{--5} = D^{--}G^{++5}\,.$$

- After passing to components, the spin 2 part of the Lagrangian reads:

$$G^{++\alpha\dot\alpha}_{(\Phi)}G^{--}_{(\Phi)\alpha\dot\alpha} + G^{++5}_{(\Phi)}G^{--5}_{(\Phi)} \quad \Rightarrow$$

$$\mathcal{L}_{(\Phi)} = -\frac{1}{4}\Big[\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\Box\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - \Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\alpha\dot\alpha}\partial^{\rho\dot\rho}\Phi_{(\rho\beta)(\dot\rho\dot\beta)}$$
$$+2\,\Phi\partial^{\alpha\dot\alpha}\partial^{\beta\dot\beta}\Phi_{(\alpha\beta)(\dot\alpha\dot\beta)} - 6\Phi\Box\Phi\Big].$$

- Invariant under $\delta\Phi_{\beta\dot\beta\alpha\dot\alpha} = \frac{1}{2}\left(\partial_{\alpha\dot\alpha}a_{\beta\dot\beta} + \partial_{\beta\dot\beta}a_{\alpha\dot\alpha}\right)\,,\ \delta\Phi = \frac{1}{4}\partial_{\alpha\dot\alpha}a^{\alpha\dot\alpha}\,.$

# $\mathcal{N} = 2$ spin 3 and higher spins

- The spin 3 triad of analytic gauge superfields is introduced as :

$$h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)\,,\ h^{++\alpha\dot\alpha}(\zeta),\ h^{++(\alpha\beta)\dot\alpha+}(\zeta),\ h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)\,,$$

and has the following transformation laws, with the analytic gauge parameters:

$$\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],$$
$$\delta h^{++\alpha\dot\alpha} = D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],$$
$$\delta h^{++(\alpha\beta)\dot\alpha+} = D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\ \delta h^{++(\dot\alpha\dot\beta)\alpha+} = D^{++}\lambda^{+(\dot\alpha\dot\beta)\alpha}\,.$$

- The bosonic physical fields in the WZ gauge are collected in

$$\begin{aligned} h^{++(\alpha\beta)(\dot\alpha\dot\beta)} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + \ldots \\ h^{++\alpha\dot\alpha} &= -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + \ldots \end{aligned}$$

- The physical gauge fields are $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}$ (spin 3), $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ (spin 2) and $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_{\gamma}$ (spin 5/2). Other fields are auxiliary. On shell, $(\mathbf{3}, \mathbf{5/2}, \mathbf{5/2}, \mathbf{2})$.

- Some residual gauge freedom can be used to put the physical bosonic gauge fields into the irreducible form

$$\Phi_{\gamma\dot\gamma(\alpha\beta)(\dot\alpha\dot\beta)} = \Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)} + \varepsilon_{\dot\gamma(\dot\alpha}\varepsilon_{\gamma(\beta}\Phi_{\alpha)\dot\beta)}\,,$$
$$C_{\gamma\dot\gamma\alpha\dot\alpha} = C_{(\gamma\alpha)(\dot\gamma\dot\alpha)} + \varepsilon_{\gamma\alpha}\varepsilon_{\dot\gamma\dot\alpha}C\,,$$

with the following gauge transformations

$$\delta\Phi_{(\alpha\gamma\beta)(\dot\alpha\dot\gamma\dot\beta)} = \partial_{(\beta(\dot\beta}a_{\alpha\gamma)\dot\alpha\dot\gamma)}\,, \quad \delta\Phi_{\alpha\dot\beta} = \frac{4}{9}\partial^{\gamma\dot\gamma}a_{(\alpha\gamma)(\dot\beta\dot\gamma)}\,,$$
$$\delta C_{(\alpha\beta)(\dot\alpha\dot\beta)} = \partial_{(\beta(\dot\beta}b_{\alpha)\dot\alpha)}, \quad \delta C = \frac{1}{4}\partial_{\alpha\dot\alpha}b^{\alpha\dot\alpha}.$$

- These are just the correct gauge transformations for the Fronsdal spin 3 fields ($\Phi_{(\alpha\beta\gamma)(\dot\alpha\dot\beta\dot\gamma)}, \Phi_{\alpha\dot\beta}$) and spin 2 fields ($C_{(\alpha\beta)(\dot\alpha\dot\beta)}, C$).

- The invariant superfield action is constructed as in the spin 2 case

$$S_{s=3} = \int d^4x d^8\theta du \left\{ G^{++(\alpha\beta)(\dot\alpha\dot\beta)} G^{--}_{(\alpha\beta)(\dot\alpha\dot\beta)} + G^{++\alpha\dot\beta} G^{--}_{\alpha\dot\beta} \right\},$$

$$G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = h^{++(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\left[h^{++(\alpha\beta)(\dot\alpha+}\bar\theta^{-\dot\beta)} - h^{++(\dot\alpha\dot\beta)(\alpha+}\theta^{-\beta)}\right],$$

$$G^{++\alpha\dot\beta} = h^{++\alpha\dot\beta} - 2i\left[h^{++(\alpha\beta)\dot\beta+}\theta^{-}_{\beta} - \bar\theta^{-}_{\dot\alpha} h^{++(\dot\alpha\dot\beta)\alpha+}\right],$$

$$D^{++}G^{--(\alpha\beta)(\dot\alpha\dot\beta)} - D^{--}G^{++(\alpha\beta)(\dot\alpha\dot\beta)} = 0\,, \; D^{++}G^{--\alpha\dot\beta} - D^{--}G^{++\alpha\dot\beta} = 0.$$

- The component spin 3 bosonic action:

$$\begin{aligned} S_{(s=3)} = \int d^4x \Big\{ & \Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \Box \Phi_{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \\ & - \frac{3}{2}\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1}\partial^{\rho\dot\rho}\Phi_{(\rho\alpha_2\alpha_3)(\dot\rho\dot\alpha_2\dot\alpha_3)} \\ & + 3\Phi^{(\alpha_1\alpha_2\alpha_3)(\dot\alpha_1\dot\alpha_2\dot\alpha_3)} \partial_{\alpha_1\dot\alpha_1}\partial_{\alpha_2\dot\alpha_2}\Phi_{\alpha_3\dot\alpha_3} - \frac{15}{4}\Phi^{\alpha\dot\alpha}\Box\Phi_{\alpha\dot\alpha} \\ & + \frac{3}{8}\partial_{\alpha_1\dot\alpha_1}\Phi^{\alpha_1\dot\alpha_1}\partial_{\alpha_2\dot\alpha_2}\Phi^{\alpha_2\dot\alpha_2} \Big\}. \end{aligned}$$

- The general case with the maximal spin **s** is spanned by the analytic gauge potentials

  $h^{++\alpha(s-1)\dot\alpha(s-1)}(\zeta), h^{++\alpha(s-2)\dot\alpha(s-2)}(\zeta), h^{++\alpha(s-1)\dot\alpha(s-2)+}(\zeta), h^{++\dot\alpha(s-1)\alpha(s-2)+}(\zeta),$

  where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot\alpha(s) := (\dot\alpha_1 \ldots \dot\alpha_s)$.
- The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1})$.

# Hypermultiplet couplings: non-conformal case

- The construction of interactions in the theory of higher spins is a very important (albeit difficult) task. There is an extensive literature related to the construction of cubic higher spin interactions (e.g., Bengtsson et al, 1983; Fradkin, Metsaev, 1991; Metsaev, 1993; Manvelyan, Mkrtchyan, Ruehl, 2010, 2011, and many others).

- Supersymmetric $\mathcal{N} = 1$ generalizations of the purely bosonic cubic vertices with matter and the corresponding supercurrents were explored in terms of $\mathcal{N} = 1$ superfields by Gates, Koutrolikos, Kuzenko, I. Buchbinder, E. Buchbinder and others.

- In JHEP 05 (2022) 104, arXiv: 2202.08196 [hep-th] we have constructed, for the first time, the off-shell manifestly $\mathcal{N} = 2$ supersymmetric cubic couplings $\mathbf{\frac{1}{2} - \frac{1}{2} - s}$ of an arbitrary higher integer superspin **s** gauge $\mathcal{N} = 2$ multiplet to the hypermultiplet matter in $4D, \mathcal{N} = 2$ harmonic superspace.

- The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S=\int d\zeta^{(-4)}\,\mathcal{L}^{+4}_{free}=-\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\mathcal{D}^{++}q^{+}_{a}\,,\,a=1,2.$$

- We reproduce the gauge higher-spin $\mathcal{N}=2$ superfields from gauging the appropriate higher-derivative rigid (super)symmetries of this free hypermultiplet action. The simplest symmetry is of zero order in derivatives, it is $U(1)$ transformation of $q^{+a}$,

$$\delta q^{+a}=-\lambda_0 Jq^{+a},\quad Jq^{+a}=i(\tau_3)^a_{\ b}q^{+b}$$

Gauging of this symmetry is just replacing $\lambda_0$ by analytic superparameter, $\lambda_0\to\lambda(\zeta)$, and in order to make the $q^{+a}$ action gauge-invariant, performing the change

$$\mathcal{D}^{++}\ \Rightarrow\ \mathfrak{D}^{++}_{(1)}=\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}\,,\quad \hat{\mathcal{H}}^{++}=h^{++}J\,,$$
$$\delta_\lambda\hat{\mathcal{H}}^{++}=[\mathcal{D}^{++},\hat{\Lambda}]\,,\quad \hat{\Lambda}=\lambda J\ \Rightarrow\ \delta_\lambda h^{++}=\mathcal{D}^{++}\lambda\,.$$

One can choose $\partial_5 q^{+a}=0$ which corresponds to massless $q^{+a}$ or $\partial_5 q^{+a}\sim mJq^{+a}$, which corresponds to massive $q^{+a}$.

- The global symmetry to be gauged in the spin 2 case is somewhat more complicated, it is of first order in derivatives

$$\delta_{rig} q^{+a} = -\hat{\Lambda}_{rig} q^{+a},$$

$$\hat{\Lambda}_{rig} = \left(\lambda^{\alpha\dot{\alpha}} - 2i\lambda^{-\alpha}\bar{\theta}^{+\dot{\alpha}} - 2i\theta^{+\alpha}\bar{\lambda}^{-\dot{\alpha}}\right)\partial_{\alpha\dot{\alpha}} + \lambda^{+\alpha}\partial^{-}_{\alpha} + \bar{\lambda}^{+\dot{\alpha}}\partial^{-}_{\dot{\alpha}} + \left(\lambda^{5} + 2i\lambda^{\hat{\alpha}-}\theta^{+}_{\hat{\alpha}}\right)\partial_{5} := \Lambda^{M}\partial_{M}\,, \quad [\mathcal{D}^{++}, \hat{\Lambda}_{rig}] = 0$$

  Five constant bosonic parameters $\lambda^{\alpha\dot{\alpha}}$, $\lambda^{5}$, four constant spinor parameters $\lambda^{\pm\hat{\alpha}} = \lambda^{\hat{\alpha}i}u^{\pm}_{i}$. We gauge these transformations.

- There are two possibilities:

$$\delta_1 q^{+a} = -\hat{\Lambda}_{(2)} q^{+a}, \qquad \hat{\Lambda}_{(2)} := \lambda^{M}\partial_{M} = \lambda^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \lambda^{+\alpha}\partial^{-}_{\alpha} + \bar{\lambda}^{+\dot{\alpha}}\partial^{-}_{\dot{\alpha}} + \lambda^{5}\partial_{5}\,,$$

$$\delta_2 q^{+a} = -\frac{1}{2}\Omega_{(2)} q^{+a}, \qquad \Omega_{(2)} := (-1)^{P(M)}\partial_{M}\lambda^{M} = \partial_{\alpha\dot{\alpha}}\lambda^{\alpha\dot{\alpha}} - \partial^{-}_{\alpha}\lambda^{+\alpha} - \partial^{-}_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}.$$

$$(\delta_1 + \delta_2)\,\mathcal{L}^{+4}_{free} = \frac{1}{2} q^{+a}[\mathcal{D}^{++}, \hat{\Lambda}_{(2)}]q^{+}_{a}.$$

- Introduce the differential operator

$$\hat{\mathcal{H}}^{++}_{(2)} = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\hat{\mu}+}\partial^{-}_{\hat{\mu}} + h^{++5}\partial_5 \,,$$

$$\delta\hat{\mathcal{H}}^{++}_{(2)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(2)}].$$

The linear in gauge superfields part of the gauge-invariant action:

$$\mathcal{L}^{+4}_{free} \;\; \to \;\; \mathcal{L}^{+4(s=2)}_{gauge} = \mathcal{L}^{+4}_{free} - \frac{1}{2}q^{+a}\hat{\mathcal{H}}^{++}_{(2)}q^{+}_{a} \,.$$

- In fact $\mathcal{L}^{+4(s=2)}_{gauge}$ can be made fully gauge invariant (and not at the linearized level) by deforming the gauge transformation law of $\hat{\mathcal{H}}^{++}_{(2)}$

$$\delta_{full}\hat{\mathcal{H}}^{++}_{(2)} = [\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(2)}, \hat{\Lambda}_{(2)}] \,.$$

- A surprising peculiarity of the superspin 3 case is that the relevant rigid two-derivative transformations to be gauged and the resulting couplings to the hypermultiplet can be consistently defined only at cost of breaking rigid $SU(2)_{PG}$ symmetry down to $U(1)$ generator which is present in all formulas. This peculiarity extends to all odd $\mathcal{N}=2$ spins.
- From the beginning one can define 4 independent transformations of $q^{+a}$, only two their fixed combinations can be compensated by the appropriate transformations of gauge superfields:

$$\delta_\lambda q^{+a} \quad = \quad -[\lambda^{\alpha\dot\alpha M}\partial_M\partial_{\alpha\dot\alpha} + \frac{1}{2}(\partial_{\alpha\dot\alpha}\lambda^{\alpha\dot\alpha M})\partial_M + \frac{1}{2}\Omega^{\alpha\dot\alpha}_{(3)}\partial_{\alpha\dot\alpha} + \frac{1}{2}\Omega_{(3)}](\tau_3)^a_{\ b}q^{+b}$$

$$\delta_\xi q^{+a} = -\xi\,\Omega_{(3)}\,Jq^{+a} = -i\xi\,\Omega_{(3)}\,(\tau_3)^a_{\ b}q^{+b}\,,$$

$$\lambda^{\alpha\dot\alpha M}\partial_M = \lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}\partial_{\beta\dot\beta} + \lambda^{(\alpha\beta)\dot\alpha+}\partial^-_\beta + \bar\lambda^{(\dot\alpha\dot\beta)\alpha+}\partial^-_{\dot\beta} + \lambda^{\alpha\dot\alpha}\partial_5\,,$$

$$\Omega_{(3)} = (\partial_{\alpha\dot\alpha}\Omega^{\alpha\dot\alpha}_{(3)})\,,\quad \Omega^{\alpha\dot\alpha}_{(3)} = (-1)^{P(M)}(\partial_M\lambda^{\alpha\dot\alpha M})\,.$$

- Defining

$$\hat{\mathcal{H}}^{++\alpha\dot{\alpha}} = h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})}\partial_{\beta\dot{\beta}} + h^{++(\alpha\beta)\dot{\alpha}+}\partial^-_{\beta} + \bar{h}^{++(\dot{\alpha}\dot{\beta})\alpha+}\partial^-_{\dot{\beta}} + h^{++\alpha\dot{\alpha}}\partial_5\,,$$

$$\Gamma^{++\alpha\dot{\alpha}} = \partial_{\beta\dot{\beta}}h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \partial^-_{\beta}h^{++(\alpha\beta)\dot{\alpha}+} - \partial^-_{\dot{\beta}}h^{++\alpha(\dot{\alpha}\dot{\beta})+},$$

$$\hat{\mathcal{H}}^{++}_{(3)} = \hat{\mathcal{H}}^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}},\ \Gamma^{++}_{(3)} = \partial_{\alpha\dot{\alpha}}\Gamma^{++\alpha\dot{\alpha}}$$

$$\delta\hat{\mathcal{H}}^{++} = [\mathcal{D}^{++}, \hat{\Lambda}_{(3)}]\,,\ \hat{\Lambda}_{(3)} = \lambda^{\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}}\,,\ \delta\Gamma^{++}_{(3)} = \mathcal{D}^{++}\Omega_{(3)}\,,$$

we obtain a gauge invariant extension of the $q^+$ action as

$$\mathcal{L}^{+4(s=3)}_{gauge} = \mathcal{L}^{+4}_{free} - \frac{1}{2}q^{+a}\left(\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(3)}J + \xi\Gamma^{++}_{(3)}J\right)q^+_a$$

- The presence of constant $\xi$ in the gauged Lagrangian shows that off shell there are 2 types of possible interactions of the $\mathcal{N}=2$ spin **3** with the hypermultiplet.
- The hypermultiplet couplings of the $\mathcal{N}=2$ gauge multiplets of higher superspins **s** can be constructed quite analogously, through the appropriate differential operators $\hat{\mathcal{H}}^{++}_{(s)}$ of the rank $\mathbf{s}-\mathbf{1}$.

# Superconformal couplings

- Free conformal higher-spin actions in $4D$ Minkowski space were pioneered by Fradkin & Tseytlin, 1985; Fradkin & Linetsky, 1989, 1991. Since then, a lot of works on (super)conformal higher spins followed (e.g., Segal, 2003, Kuzenko, Manvelyan, Theisen, 2017; Kuzenko, Ponds, Raptakis, 2023; ......).
- $\mathcal{N} = 2, 4D$ case was elaborated in Kuzenko, Raptakis, 2021 using the notion of $\mathcal{N} = 2$ conformal superspace. The corresponding Noether couplings to on-shell hypermultiplet were constructed there.
- Quite recently (Buchbinder, Ivanov, Zaigraev, to appear), we extended our previous results on the off-shell $\mathcal{N} = 2, 4D$ higher spins and their hypermultiplet cubic coupling in HSS to the superconformal case. Rigid $\mathcal{N} = 2, 4D$ superconformal symmetry realized in HSS (Galperin, Ivanov, Ogievetsky, Sokatchev, 1985, 1987) plays a crucial role in fixing the structure of this theory.
- $\mathcal{N} = 2, 4D$ SCA preserves harmonic analitycity and is a closure of the rigid $\mathcal{N} = 2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha} i} u^+_i \,, \; \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i \left( \epsilon^{\alpha i} \bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha} \bar{\epsilon}^{\dot{\alpha} i} \right) u^-_i \,, \hat{\alpha} = (\alpha, \dot{\alpha}) \,,$$

$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}} k_{\beta\dot{\beta}} \theta^{\hat{\beta}} \,, \; \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha} \,, \; \delta_k u^{+i} = (4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}}) u^{-i} \,.$$

- What about the conformal properties of various analytic higher-spin potentials? No problems with the spin **1** potential $V^{++}$:

$$\delta_{sc} V^{++} = -\hat{\Lambda}_{sc} V^{++}\,, \quad \hat{\Lambda}_{sc} := \lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+} + \lambda^{++}_{sc}\partial^{--}$$

- The cubic vertex $\sim q^{+a}V^{++}Jq^{+}_{a}$ is invariant up to total derivative if

$$\delta_{sk} q^{+a} = -\hat{\Lambda}_{sc} q^{+a} - \frac{1}{2}\Omega q^{+a}\,, \quad \Omega := (-1)^{P(M)}\partial_M \lambda^M$$

 Moreover, this vertex is invariant under arbitrary analytic superdiffeomorphisms, $\Lambda_{sk} \to \Lambda(\zeta)$.

- Situation gets more complicated for **s** $\geq 2$. Requiring $\mathcal{N} = 2$ gauge potentials or **s** $= 2$ to be closed under $\mathcal{N} = 2$ SCA necessarily leads to

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,,$$
$$\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^{-}_{\alpha} + h^{++\dot{\alpha}+}\partial^{-}_{\dot{\alpha}} + h^{(+4)}\partial^{--}$$
$$\delta_{k_{\alpha\dot{\alpha}}} h^{(+4)} = -\hat{\Lambda} h^{(+4)} + 4ih^{++\alpha+}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}} + 4i\theta^{+\alpha}h^{++\dot{\alpha}+}k_{\alpha\dot{\alpha}}$$

 It is impossible to avoid introducing the extra potential $h^{(+4)}$ for ensuring conformal covariance. The extended set of potentials encompasses $\mathcal{N} = 2$ **Weyl multiplet** ($\mathcal{N} = 2$ conformal SG gauge multiplet).

- For $\mathbf{s} \geq 3$ the gauge-covariantization of the free $q^{+a}$ action requires adding the gauge superfield differential operators of rank $\mathbf{s-1}$ in $\partial_M$,

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)} (J)^{P(s)}, \quad P(s) = \frac{1 + (-1)^{s-1}}{2}$$

- For $\mathbf{s} = 3$:

$$\hat{\mathcal{H}}_{(s=3)} = h^{++MN} \partial_N \partial_M + h^{++}, \quad h^{++MN} = (-1)^{P(M)P(N)} h^{++NM}$$

- $\mathcal{N} = 2$ SCA mixes different entries of $h^{++MN}$, so we need to take into account all these entries, as distinct from non-conformal case where it was enough to consider, e.g., $h^{++\alpha\dot{\alpha}M}$.
- The spin $\mathbf{3}$ gauge transformations of $q^{+a}$ and $h^{++MN}$ leaving invariant the action $\sim q^{+a}(D^{++} + \kappa_3 \hat{\mathcal{H}}_{(s=3)}) q^+_a$ are

$$\delta_\lambda^{(s=3)} q^{+a} = -\frac{\kappa_3}{2} \{\hat{\Lambda}^M, \partial_M\}_{AGB} J q^{+a} - \frac{\kappa_3}{4} \{\Omega^M, \partial_M\}_{AGB} J q^{+a},$$

$$\delta_\lambda^{(s=3)} \hat{\mathcal{H}}^{++}_{(s=3)} = \frac{1}{2} \left[\mathcal{D}^{++}, \{\hat{\Lambda}^M, \partial_M\}_{AGB}\right],$$

$$\hat{\Lambda}^M := \sum_{N \leq M} \lambda^{MN} \partial_N, \ \Omega^M := \sum_{N \leq M} (-1)^{[P(N)+1]P(M)} \partial_N \lambda^{NM},$$

$$\{F_1, F_2\}_{AGB} = [F_1, F_2], \quad \{B_1, B_2\}_{AGB} = \{B_1, B_2\}.$$

- All the potentials except $h^{++\alpha\dot{\alpha}M}$ can be put equal to zero using the original extensive gauge freedom:

$$S^{(s=3)}_{int|fixed} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\, q^{+a} h^{++\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}} J q^+_a\,. \tag{1}$$

- In such a gauge one is led to accompany the superconformal transformations by the proper compensating gauge transformations in order to preserve the gauge, so the final SC transformations are nonlinear in $h^{++M\alpha\dot{\alpha}}$.
- Using the linearized gauge transformations of $h^{++\alpha\dot{\alpha}M}$

$$\begin{aligned}
\delta_\lambda h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= \mathcal{D}^{++}\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 4i\lambda^{+(\alpha\beta)(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})} + 4i\theta^{+(\alpha}\bar{\lambda}^{+\beta)(\dot{\alpha}\dot{\beta})},\\
\delta_\lambda h^{++(\alpha\beta)\dot{\alpha}+} &= \mathcal{D}^{++}\lambda^{+(\alpha\beta)\dot{\alpha}} - \lambda^{++(\alpha\dot{\alpha}}\theta^{+\beta)},\\
\delta_\lambda h^{++(\dot{\alpha}\dot{\beta})\alpha+} &= \mathcal{D}^{++}\lambda^{+(\dot{\alpha}\dot{\beta})\alpha} - \lambda^{++\alpha(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})},\\
\delta_\lambda h^{(+4)\alpha\dot{\alpha}} &= \mathcal{D}^{++}\lambda^{++\alpha\dot{\alpha}} - 4i\bar{\theta}^{+\dot{\alpha}}\lambda^{+\alpha++} + 4i\theta^{+\alpha}\lambda^{+\dot{\alpha}++},
\end{aligned}$$

we can find WZ gauge for the spin 3 gauge supermultiplet

$$\begin{aligned}
h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= -4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}_{\rho\dot{\rho}} + (\bar{\theta}^+)^2\theta^+\psi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i\\
&\quad + (\theta^+)^2\bar{\theta}^+\bar{\psi}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})ij}u^-_i u^-_j\,,\\
h^{++(\alpha\beta)\dot{\alpha}+} &= (\theta^+)^2\bar{\theta}^+_{\dot{\nu}} P^{(\alpha\beta)(\dot{\alpha}\dot{\nu})} + (\bar{\theta}^+)^2\theta^+_\nu T^{(\alpha\beta\nu)\dot{\alpha}} + (\theta^+)^4\chi^{(\alpha\beta)\dot{\alpha}i}u^-_i\,,\\
h^{(+4)\alpha\dot{\alpha}} &= (\theta^+)^2(\bar{\theta}^+)^2 D^{\alpha\dot{\alpha}}.
\end{aligned}$$

- In the physical sector we are left with the spin $3$ gauge field, spin $5/2$ doublet, "higher-spin" $SU(2)$ gauge field and the conformal graviton:

$$\Phi_{\rho\dot{\rho}}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}, \quad \psi_{\hat{\mu}}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}, \quad V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})(ij)}, \quad P^{(\alpha\beta)(\dot{\alpha}\dot{\nu})}. \tag{2}$$

- The remaining fields,

$$T^{(\alpha\beta\gamma)\dot{\alpha}}, \quad \chi^{(\alpha\beta)\dot{\alpha}i}, \quad D^{\alpha\dot{\alpha}}, \tag{3}$$

  are auxiliary.
- The sum of conformal spin $2$ and spin $3$ actions

$$S = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\right) q^+_a\,. \tag{4}$$

  is invariant with respect to the (properly modified) spin $\mathbf{3}$ transformations to the leading order in $\kappa_3$ and to any order in $\kappa_2$. This means that the cubic vertex $(\mathbf{3}, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{1}}{\mathbf{2}})$ constructed is invariant under the gauge transformations of conformal $\mathcal{N} = 2$ supergravity. In the component approach, after elimination of the auxiliary fields, one should recover the superconformal action of the spin $\mathbf{3}$ supermultiplet on *generic* $\mathcal{N} = 2$ Weyl supergravity background.
- The whole consideration can be generalized to the general integer higher-spin $s$ case.

# Summary and outlook

The theory of $\mathcal{N} = 2$ higher spins $s \geq 3$ opens a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N} = 2$ theories with the maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories is the harmonic Grassmann analyticity.

**Under way**:

- Higher spin $\mathcal{N} = 2$ superconformal generalized Weyl actions?
- $\mathcal{N} = 2$ supersymmetric half-integer spins?
- An extension to AdS and other conformally-flat backgrounds?
- Quantum calculations using a generalization of the background $\mathcal{N} = 2$ HSS methods to higher spins. Finding out possible induced higher-spin superconformal actions.
- From the linearized theory to its full nonlinear version? The latter is known at present only for $s \leq 2$ ($\mathcal{N} = 2$ super Yang - Mills and $\mathcal{N} = 2$ supergravities). This problem will seemingly require accounting for ALL higher $\mathcal{N} = 2$ superspins simultaneously. New supergeometries?

THANK YOU FOR ATTENTION!