On characteristic boundary value problem and polyhomogeneous spacetimes

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Overview

1. Introduction
   - Characteristic hypersurfaces
   - Bondi-Sachs formalism
2. Polyhomogenous spacetimes
   - Polyhomogeneous spacetimes
3. Examples from ED and GR
   - ED with null cosmic string

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Characteristic initial value problem

Before 1960 it was know that linear pertubations h_{ab} of the Minkowski metric $\eta_{ab} = diag(-1,1,1,1)$ obeyed the wave equation $(c = G = 1)$

$$\left(-\frac{\partial^2}{\partial t^2} + \delta_{ij}\frac{\partial^2}{\partial x^i \partial x^j}\right) h_{ab} = 0,$$

where the standard Cartesian coordinates $x^i = x^1, x^2, x^3$ satisfy the harmonic coordinate condition to linear order. The gauge freedom of these linear perturbations raised doubts about the physical properties of gravitational waves.

The hypersurfaces $u = t - r = const$, $v = t + r = const$, with $r^2 = \delta_{ij} y^i y^j$, are characteristic hypersurfaces of the wave equation, i.e. they are hypersurfaces along which wavefronts can travel.

- A second order hyperbolic PDE allows a well-posed Cauchy problem on initial space-like hypersurfaces. If the initial hypersurfaces is null (characteristic), the standard Cauchy data, fields and their first derivatives, are not independent. It will be discussed later.

Characteristic surface = null surface

These characteristic hypersurfaces are also null hypersurfaces, i.e. their normals, $k_a = -\nabla_a u$ and $n_a = -\nabla_a v$ are null,

$$\eta^{ab} k_a k_b = \eta^{ab} n_a n_b = 0$$

• A peculiar property of null hypersurfaces that their normal direction is also tangent to the hypersurface, $k^a = \eta^{ab} k_b$ is tangent to $u = const$.
The curves tangent to k^a are null geodesics, called null rays, and generate the $u = const$ outgoing null hypersurfaces.

For differential geometry of null surfaces see Gourgoulhon and Jaramillo (2006).

• Bondi used such a family of outgoing null rays forming these null hypersurfaces to build spacetime coordinates for describing outgoing gravitational waves.

Bondi-Sachs metric

Sachs (1962): Given any normal-hyperbolic Riemannian manifold with line element ds^2 and in it any point P, there exists at least one set of coordinates $u = x^0, t = x^1, \theta = x^2, \phi = x^3$ such that in a finite neighborhood of P,

$$ds^2 = g_{ab}dx^a dx^b = -\frac{V}{r}e^{2\beta}du^2 - 2e^{2\beta}dudr + r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du) \quad (\star),$$

where $A, B, \dots = 2, 3$; h_{AB} is a conformal metric with two degrees of freedom, $\det(h_{AB}) = b(u, \theta, \phi)$.

$(\star)$ holds if and only if the coordinates u, r, θ, ϕ have the following geometric properties
(i) the hypersurfaces $u = const$ are everywhere tangent to the local lightcone;
(ii) r varies along the light (null) rays; r is chosen to be an areal coordinate, such that $det[g_{AB}] = r^4 det[q_{AB}]$, q_{AB} is the unit sphere metric.
(iii) the angular coordinates x^A, (A,B,...=2,3) are constant along the null rays

V, β, U^A and h_{AB} are any six functions of the coordinates.

Asymptotic flatness

The approach of Bondi, Penrose ans Sachs imposed some restrictions on the space-time properties.

- Space-time should be asymptotically flat, possessing the Euclidean topology far from isolated source.

In the asymptotic inertial frame, often referred to as a Bondi frame, the metric approaches the Minkowski metric at null infinity $\mathcal{J}^+$ [a null surface with $(r=\infty, u, x^A)$], so that

$$\lim_{r\to\infty}\beta = \lim_{r\to\infty} U^A = 0, \quad \lim_{r\to\infty}\frac{V}{r} = 1, \quad \lim_{r\to\infty} h_{AB} = q_{AB}.$$

Thus the Minkowski metric in outgoing null spherical coordinates (u, r, x^A) corresponding to the flat space version of the Bondi-Sachs metric is

$$ds^2 = g_{ab}dx^a dx^b = -du^2 - 2dudr + r^2 q_{AB}dx^A dx^B.$$

These restrictions were based on an intuitive analogy with a retarded solution for a point source in electrodynamics, which can be represented as expansion in multipoles.

The Bondi-Sachs solution

In Bondi-Sachs approach the Einstein equations,

$$R_{ab} - \frac{1}{2}g_{ab}R = 8\pi T_{ab},$$

form a hierarchy (4 hypersurface equations, 2 standard equations, 1 trivial equation, and 3 supplementary conditions).

• For integrating it is assumed that matter satisfies the divergence-free condition $\nabla_b T_a^b = 0$, the matter sources are confined to a compact region. The solution has an asymptotic $1/r$ expansion in an asymptotic inertial frame.

• This ansatz of a 1/r-expansion of the metric leads to the peeling property of the Weyl tensor in the spin-coefficient approach Newman and Penrose (1962)

The projections Ψ_i of the Weyl tensor onto the tetrad, corresponding to Bondi-Sachs coordinates with radial coordinate r chosen as an affine parameter along the outgoing null geodesics, should have the asymptotics:

$$\Psi_i = C_i(u, \theta, \phi) r^{i-5} + \mathcal{O}(r^{i-6}), \quad i = 0..4.$$

• The conformal 2-metric h_{AB} on an initial null hypersurface N_0, $u = u_0$, is assumed to have the asymptotic 1/r expansion

$$h_{AB}(u_0, r, x^C) = q_{AB} + \frac{c_{AB}(u_0, x^E)}{r} + \frac{d_{AB}(u_0, x^E)}{r^2} + \dots$$

• The $1/r$ coefficient of the conformal 2-metric h_{AB} for retarded times $u \in [u_0, u_1], u_1 > u_0$,

$$c_{AB}(u, x^C) := \lim_{r\to\infty} r(h_{AB} - q_{AB})$$

describes the time dependence of the gravitational radiation. Its retarded time derivative

$$N_{AB} = \frac{1}{2}\partial_u c_{AB}(u, x^E),$$

is called the news tensor, and it determines the energy flux of gravitational radiation.

•The BS formalism provided the first convincing evidence that that mass loss due to gravitational radiation is a nonlinear effect of general relativity and that the emission of gravitational waves from an isolated system is accompanied by a mass loss from the system.

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Polyhomogeneous spacetimes

In general case, the approximate solutions of Einstein equations near $\mathcal{J}^+$ admit not only power, but also logarithmic terms of the form $\ln^m r/r^n$. Such spaces are called polyhomogeneous, [Valiente Kroon (1999b)] or logarithmically asymptotically flat [Winicour (1985)].

The peeling property was historically related to the "Outgoing Radiation Condition" [Valiente Kroon (1999a)] being an analog of the Sommerfeld condition in electrodynamics. Bondi, Sachs and Penrose used Outgoing Radiation Condition in order to eliminate logarithmic terms in the asymptotics for the metric, as far as non-analyticity was erroneously associated with the presence of an incoming radiation.

Subsequent analysis showed that Bondi-Sachs-Penrose requirement for the metric asymptotics is excessively strict. Such solutions may exist, but require specific initial data.

The Bondi-Sacks-Penrose approach can be adapted accordingly. Bondi mass and the outgoing energy flux may be related with logarithmic terms in the expansion for metrics.

ED with null cosmic string

In [Fursaev and Pirozhenko (2023)] we studied perturbations of EM fields of point electric charges induced by close passage of a null cosmic string. Then we generalized this result to point magnetic-dipole-like sources. The asymptotics for angular components of the vector potential at at future null infinity looks as

$$A_B(r,u,\Omega) \simeq a_B(u,\Omega) + b_B(\Omega)\ln r/\varrho + O(r^{-1}\ln r) \quad , \tag{1}$$

where u is the retarded time, vector fields a_B, b_B are in the tangent space of S^2, and ϱ is a dimensional parameter related to the approximation. The asymptotic holds if r is large enough with respect to u and with respect to the impact parameter between the string and the source. Equation (1) yields a finite energy flux at large r

$$\lim_{r\to\infty} \partial_t E(R,t) = \int d\Omega\; \dot{a}_A \dot{a}^A \quad ,$$

where $\dot{a}_A \equiv \partial_U a_A$, and index A is risen with the help of the metric on S^2. The logarithmic term in (1) appears since the Sommerfeld radiation condition is violated in the presence of null strings. This term, however, does not affect the flux.

Null strings (massless, tensionless)

From a massive cosmic string at rest along z-axis

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\Theta^2, \quad r^2 = x^2 + y^2$$

$\longrightarrow$ Aichelburg-Sexl boost

$$\cosh\chi = (1 - v^2/c^2)^{-1/2} \to \infty, \quad E = mc^2 \cosh\chi \to \mathit{finite}$$

$\longrightarrow$ Kerr-Schield metric

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi G\varepsilon$$

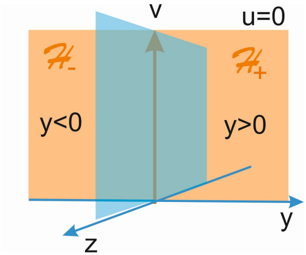
ε - energy per unit length, $u = t - x$, $v = t + x$.

C. Barrabes, P.A. Hogan, W. Israel, Phys.Rev. D66 (2002) 025032.

Classical ED with a null cosmic string

We consider a null cosmic string stretched along z-axis and moving along x-axis. The space-time is locally Minkowsky $R^{1,3}$, and in the light-cone coordinates the metric reads

$$ds^2 = -dvdu + dy^2 + dz^2, \quad v = t + x,\ u = t - x.$$



- The string world surface S: $u = y = 0$
- The string event horizon $\mathcal{H}$: $u = 0$

$\mathcal{H}_+$: $u = 0, y > 0$ (right),
$\mathcal{H}_-$: $u = 0, y < 0$ (left)

The trajectories of particles and light rays at $u < 0$ and $u > 0$ are called ingoing and outgoing respectively.

Null rotations

A parallel transport of a vector V along a closed contour around a null string with the world-sheet $u = y = 0$ results in a null rotation,

$$V' = M(\omega)V, \quad \omega \equiv 8\pi GE \quad .$$

M. van de Meent, Geometry of Massless Cosmic Strings, Phys. Rev. D87 (2013) 025020

Null rotations $(x')^\mu = M^\mu_{\ \nu}(\omega)\, x^\nu$ are defined as

$$u' = u \quad , \quad v' = v + 2\omega y + \omega^2 u, \; y' = y + \omega u, \; z' = z, \quad (\star)$$

Coordinate transformations $M(\omega)$ at $u = 0$ belong to Carroll group [Duval et al. (2014b)] and induce the change of coordinates on $\mathcal{H}$

$$x^a = C^a_{\ b}(\omega)\bar{x}^b \quad \text{or} \quad \mathbf{x} = \bar{\mathbf{x}} + 2\omega y \mathbf{q} \quad , \quad q^a = \delta^a_v \quad ,$$

where $\mathbf{x} \equiv \{v, y, z\}$. Matrices $C^a_{\ b}$ can be defined as $C^a_{\ b} = M^a_{\ b}$.

Initial data on the string horizon

The initial data on the string horizon $\mathcal{H}$ are related to the ingoing data via null rotations taken at $u = 0$. To describe outgoing trajectories, one can introduce either R chart with a cut on the left to the string, $\mathcal{H}_-$ $(u = 0, y < 0)$, or L-chart with a cut on the right to the string, $\mathcal{H}_+$ $(u = 0, y > 0)$. These descriptions are equivalent due to Lorenz invariance.

Let x^μ and $\bar{x}^\mu$ be the coordinates above and below the horizon. In the R-charts the transition conditions on $\mathcal{H}$ are

$$x^\mu = \bar{x}^\mu \,|_{\mathcal{H}_+} \;, \quad x^\mu = M^\mu_{\ \nu}(\omega)\,\bar{x}^\nu \,|_{\mathcal{H}_-} \quad .$$

- At $y < 0$ the coordinate transformations are reduced to

$$v = \bar{v} + 2\omega y \quad , \quad y < 0 \quad .$$

- 4-velocities of particles and light rays change when crossing $\mathcal{H}$. On the R-charts the 'right' trajectories $(y > 0)$ behave smoothly across the horizon, while the 'left' trajectories $(y < 0)$ are shifted along the v coordinate and change their direction under the null rotation.

Initial data for EM field (R-chart)

Let $\bar{A}_\mu$ be a solution to the problem $\partial_\mu \bar{F}^{\mu\nu} = \bar{j}^\nu$ at $u < 0$. We need to solve $\partial_\mu F^{\mu\nu} = j^\nu$ at $u > 0$ with some initial conditions on null hypersurface $\mathcal{H}$.

The currents are continuous on $\mathcal{H}$ and at $u > 0$ and $u < 0$ they are related on as

$$j^\mu(\mathbf{x}) \,|_{\mathcal{H}_+} = \bar{j}^\mu(\mathbf{x}) \quad , \quad j^\mu(\mathbf{x}) \,|_{\mathcal{H}_-} = M^\mu_{\ \nu}(\omega)\bar{j}^\nu(\bar{\mathbf{x}}) \quad ,$$

$$\bar{\mathbf{x}} = \mathbf{x} - 2\omega y \mathbf{q} \quad , \quad q^i = \delta^i_v \ , \quad \mathbf{x} \equiv \{v, y, z\}.$$

Indices $a, b, ..$ enumerate coordinates v, y, z on $\mathcal{H}$. Indices a, b cannot be risen or lowered since the metric of $\mathcal{H}$ is degenerate. $M_a^{\ c}(\omega)$, $M^a_{\ c}(\omega)$ coincide with corresponding components of $M_\mu^{\ \nu}(\omega)$.

The conditions on three components A_b, are

$$a_b(\mathbf{x}) = A_b(x) \,|_{\mathcal{H}} \quad , \quad b = v, y, z \quad ,$$

$$a_b(\mathbf{x}) = \bar{a}_b(\mathbf{x}) \,|_{\mathcal{H}_+} \quad , \quad a_b(\mathbf{x}) = M_b^{\ c}(\omega)\bar{a}_c(\bar{\mathbf{x}}) \,|_{\mathcal{H}_-} \quad ,$$

$$\bar{a}_b(\mathbf{x}) = \bar{A}_b(x) \,|_{\mathcal{H}} \quad , \quad b = v, y, z \quad .$$

Characteristic initial value problem for Maxwell equations

For the Maxwell equations the canonical coordinates and momenta can be determined by using the Hamilton-Jacobi method from the variation of the action with $\mathcal{H}$ as a boundary,

$$A_b \,|_{\mathcal{H}} = a_b \quad , \quad F^{ub} \,|_{\mathcal{H}} = \pi_b \quad , \quad b = v, y, z \quad .$$

The data are subject to the constraint

$$\partial_v \pi_v + \partial_x \pi_x + \partial_y \pi_y = -j^u \,|_{\mathcal{H}} \quad . \qquad (\star)$$

The momenta $\pi_y = 2F_{vy}$, $\pi_z = 2F_{vz}$ are determined by the initial data a_b. The momentum $\pi_v = 4F_{uv}$ is fixed by $(\star)$.

Since there is the gauge freedom in definition of π_y, π_z, the initial value problem on $\mathcal{H}$ requires 2 independent data, twice less than that on space-like hypersurface.

If we fix the Lorentz gauge the initial data can be formulated as $A_\mu \,|_{\mathcal{H}} = a_\mu$, where a_u should be determined by the gauge condition. The remaining gauge freedom then leaves 2 independent data. Constraint follows from the gauge condition and equation for the v component.

The solution

We suppose that charged particles do not cross the left part of the horizon, $\bar{j}^\mu = 0$ on $\mathcal{H}_-$. This implies that $j^\mu = \bar{j}^\mu$ on $\mathcal{H}$, that is the current in the R-chart is not affected by the string. One can write a solution in the form

$$A_\mu(x) = \bar{A}_\mu(x) + A_{S,\mu}(x) \ ,$$

$\bar{A}_\mu$ as a solution in the absence of the string, $-\infty < u < \infty$,
$A_{S,\mu}$ is a perturbation caused by the string.

$A_{S,\mu}$ is a solution to a homogeneous characteristic initial value problem

$$\partial_\mu F_S^{\mu\nu} = 0 \ , \quad A_{S,b}(x) \, |_{\mathcal{H}} = a_{S,b}(\mathbf{x}) \ ,$$

$$a_{S,b}(\mathbf{x}) \, |_{\mathcal{H}_+} = 0 \ , \quad a_{S,b}(\mathbf{x}) \, |_{\mathcal{H}_-} = M_b{}^c(\omega) \bar{a}_c(\bar{\mathbf{x}}) - \bar{a}_b(\mathbf{x}) \ ,$$

where $F_{S,\mu\nu} = \partial_\mu A_{S,\nu} - \partial_\nu A_{S,\mu}$.

The properties of the perturbation $A_{S,\mu}$

i) vanishes in the limit $\omega \to 0$, where ω is the energy of the string;
ii) depends on the choice of the source j_μ through initial data ;
ii) can be written as

$$A_{S,\mu}(x) = \mathcal{A}^{\omega}_{\mu}(x) - \mathcal{A}_{\mu}(x) \quad ,$$

$$\mathcal{A}^{\omega}_{\mu}(x') = M_{\mu}^{\ \nu}(\omega)\mathcal{A}_{\nu}(x) \quad , \quad (x')^{\mu} = M^{\mu}_{\ \nu}(\omega)\, x^{\nu} \quad ,$$

where $\mathcal{A}_\mu(x)$ is a solution to homogeneous problem $\partial_\mu \mathcal{F}^{\mu\nu} = 0$ with the following initial data on $\mathcal{H}$

$$\mathcal{A}_b(\mathbf{x}) \mid_{\mathcal{H}_+} = 0 \quad , \quad \mathcal{A}_b(\mathbf{x}) \mid_{\mathcal{H}_-} = \bar{a}_b(\mathbf{x}) \quad .$$

We use the Lorentz gauge condition $\partial A = 0$ since it is invariant under null rotations and can be imposed globally on cosmic string space-time.

The Maxwell equations for the perturbation caused are reduced to

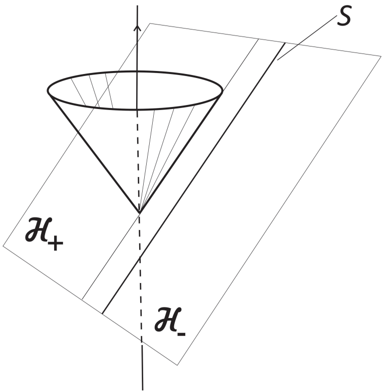
$$\Box A_{S,\mu} = 0 \quad , \quad \partial A_S = 0 \quad .$$

The component A_u follows from condition $\partial A = 0$.

The system

We consider perturbations $A_{S,\mu}$, which are solutions to the Maxwell equations for point-like sources of electric and magnetic types.

We suppose that the source is at rest at a point with coordinates $x_o = z_o = 0, y_o = a > 0$. Since the string trajectory is $x = t$, $y = 0$, we interpret a as an impact parameter between the string and the source. Electric or magnetic source crosses the string horizon $\mathcal{H}_+$. S is the world surface of the string.



The point charges

Here the sources of the electric type are electric charges with the current

$$j_0 = e\delta^{(3)}(\vec{x} - \vec{x}_o) \quad , \quad j_i(x) = 0 \quad i = 1, 2, 3 \ .$$

In the absence of the string the field is

$$\bar{A}_0(x) = e\phi(x) \ , \quad \bar{A}_i(x) = 0, \quad \phi(x, y, z) = -\frac{1}{4\pi}\frac{1}{\sqrt{x^2 + (y - a)^2 + z^2}} \ .$$

The sources of the magnetic type are described by the current

$$j_0 = 0, \quad j_i(x) = \varepsilon_{ijk} M_j \, \partial_k \delta^{(3)}(\vec{x} - \vec{x}_o) \ .$$

where M_i is a magnetic moment. In the absence of a string the field is

$$\bar{A}_0(x) = 0, \quad \bar{A}_i(x) = \varepsilon_{ijk} M_j \partial_k \phi. \quad (\star)$$

We consider $(\star)$ as an approximation for a EM field of a body of with a magnetic moment **M**. Under certain assumptions this approximation can be used to describe the EM field of pulsars.

Energy flux

The energy of the EM field inside the sphere of the radius R with the center at the point $x^i = 0$ is

$$E(R,t) = \int_{r<R} d^3x \ T_{00} \quad ,$$

where T^{μ}_{ν} is the stress-energy tensor of the EM field,

$$T^{\mu}_{\nu} = -F^{\mu A} F_{\nu A} + \delta^{\mu}_{\nu} \frac{1}{4} F^{AB} F_{AB} \quad .$$

The energy density T_{00}, is measured in the frame of reference where the charge is at rest. The conservation law implies that

$$\partial_t E(R,t) = R^2 \int d\Omega \ T^r_U \quad .$$

Energy flux at $r \gg a$ and $r \gg U$

We have derived the following asymptotics of the solution at $r \gg a$ and $r \gg U$

$$A_{S,\mu}(r,U,\Omega) \simeq \frac{1}{r}(a_\mu(U,\Omega) + b_\mu(\Omega)\ln r/\varrho) + \mathcal{O}(r^{-2}) \quad , \quad \mu = U, r \quad ,$$

$$A_{S,B}(r,U,\Omega) \simeq a_B(U,\Omega) + b_B(\Omega)\ln r/\varrho + \mathcal{O}(r^{-2}) \quad , \quad B = \theta, \varphi \quad ,$$

ϱ is a dimensional parameter related to the approximation.

The T^r_U component of the EMT is

$$T^r_U = F_{U\mu}F^{r\mu} \simeq \frac{1}{r^2}\gamma^{AB}\dot{a}_A\dot{a}_B \quad , \quad \dot{a}_A \equiv \partial_U a_A,$$

and the energy flux at large r is finite:

$$\lim_{r\to\infty} \partial_t E(R,t) = \int d\Omega\ \gamma^{AB}\dot{a}_A\dot{a}_B = \int d\Omega \left(\dot{\vec{a}}^2 - (\dot{\vec{a}}\cdot\vec{n})^2\right)$$

The r.h.s. is given for components in Minkowsky coordinates, $n^i = x^i/r$. The calculation of the flux was our main goal.

Some remarks about the energy flux

- The energy flux at $\mathcal{I}^+$, is determined only by the angular components of the perturbation A_S.
- $\bar{A}$ contributes to $F_{\mu\nu}$ but does not contribute to the flux, since $\bar{A}$ is static. Thus, the energy flux at $\mathcal{I}^+$, is determined only by the angular components of the perturbation A_S.

By varying the parameter ϱ one adds to a_A some term proportional to b_A. This addition does not affect the flux since b_A are static.

- Our solution does not satisfy the Sommerfeld radiation condition, as we consider the problem with initial data at an infinite null hypersurface *A. Sommerfeld, Jahresber. d. deutsch. math. Ver., 21, 309 (1912), A.N. Tikhonov, A.A. Samarsky, ZhETP 18, 243 (1948)*

The power of the radiation emitted to $\mathcal{J}^+$

$$\partial_t E(R,t) = \int d\Omega\, \gamma^{AB} \dot{a}_A \dot{a}_B = \int d\Omega f_E(U,\Omega) \quad ,$$

- For $U \to \infty$ the flux density vanishes as $(U/a)^{-2}$.
- The peak power is near $U = 0$. At $U = 0$, the flux density reduces to

$$f_E(0,\Omega) = \frac{e^2}{(2\pi)^4} \frac{\omega^2}{a^2} \frac{(1-n_y^2)}{2(1-n_x) - n_y^2}.$$

Here $n_x = \cos\theta, n_y = \sin\theta \sin\varphi$. The angular distribution has a pole-like singularity at $n_x = 1$. At $U = 0$ this point lies on the string world-sheet, and our approximation is not valid.

- For small U the energy flux is focused in the direction of string motion ,

$$f_E(U)_{n_x \to 1} = \frac{e^2}{16\pi^4} \frac{\omega^2}{U^2}, \quad f_E(U)_{n_x \to -1} = \frac{e^2}{16\pi^4 a^2} \frac{4\omega^2}{(4+(U/a)^2)^2},$$

thus the energy fl follows the string.

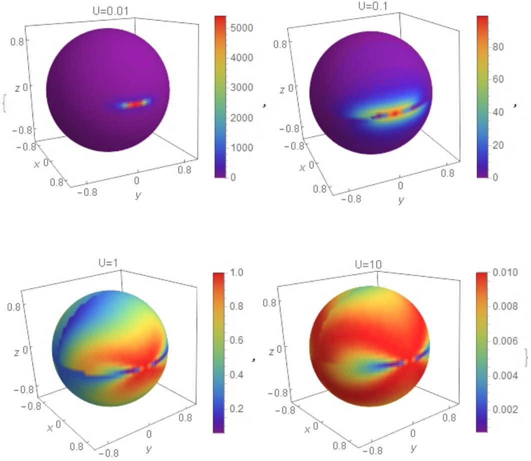


Рис. 1: The flux density from an electric source observed in 4π geometry for different values of the Bondi coordinate $U = t - r$, $U = 0.01, 0.1, 1, 10$.

An estimate of EM radiation due to null strings

- For an electrically charged source the power of the radiation pulse at the peak is

$$\partial_t E(R,t) \sim \frac{e^2}{(2\pi)^4}\,\frac{\omega^2}{a^2} \quad .$$

If $\omega \simeq 10^{-12}$ (this is an upper limit on the energy of tensile cosmic strings from stochastic gravitational-wave background), assume that $e^2/(4\pi) = 1/137$ the peak power has the order of magnitude 10^{-17} W

- For the radiation from magnetic charge,

$$\partial_t E(R,t) \sim \frac{e^2}{(2\pi)^4}\,\frac{\omega^2 M^2}{a^4} \quad .$$

Consider a magnetar with magnetic field $\sim 10^{11}\ T$. Then, a string passing in the near zone at a distance $a = 10^2$ km, carries away an electromagnetic pulse

$$\dot{E} \sim 10^{17}\ W\ .$$

For comparison the radiation power of the Sun reaches $10^{26} W$.

Conclusion

- We have studied electrodynamics in a the space-time with a straight null cosmic string. The problem appeared to be characteristic problem with initial data imposed at the string horizon. We considered two cases of point-like sources crossing the string horizon: sources with an electric charge and sources with a magnetic moment.
- We have shown that null cosmic strings disturb electric fields of charged sources and produce electromagnetic (EM) pulses.
- An analogous effect exists in gravity: perturbations of gravitational fields of massive sources caused by null cosmic strings are radiated away in a form of gravitational wave pulses.
- Perturbations generated by the strings are solutions to a characteristic Cauchy problem where the initial data are set on the string horizon $\mathcal{H}$ and take into account the planar supertranslations.
- Near the future null infinity the resulting geometry, sourced by the sting and a point-like mass, belongs to a class of so called polyhomogeneous spacetimes.

D.V. Fursaev, E.A. Davydov, V.A. Tainov, I.G. Pirozhenko, work in progress

Thank you!

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